Mechanics on our planet: Ice sheets, Earthquakes, Hydraulic fractures
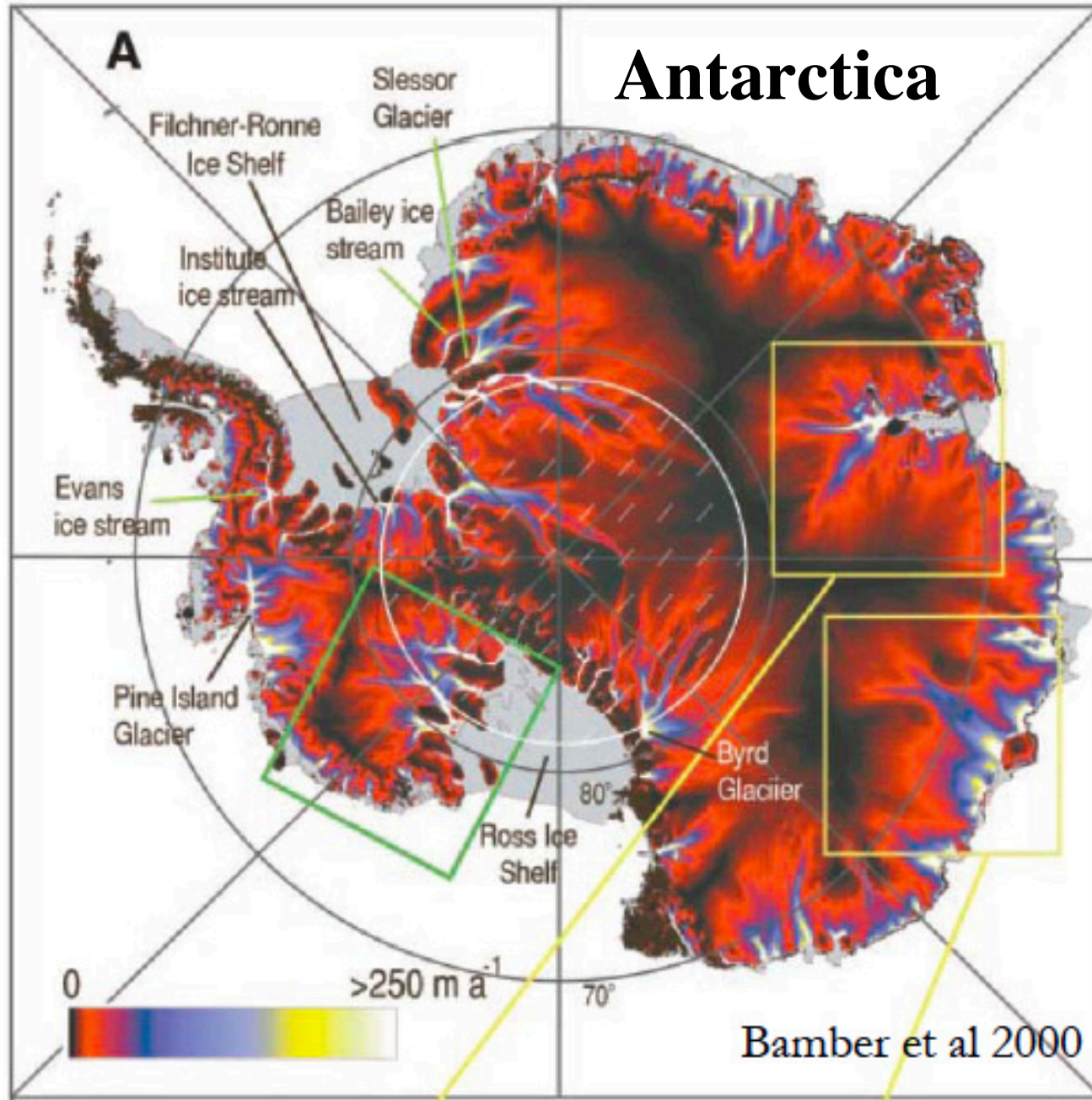
James R. Rice (*Harvard*)

Ice sheet collaborators:
Thibaut Perol (*Harvard*), John D. Platt (*Harvard*),
Jenny Suckale (*Harvard*), Victor C. Tsai (*Caltech*)
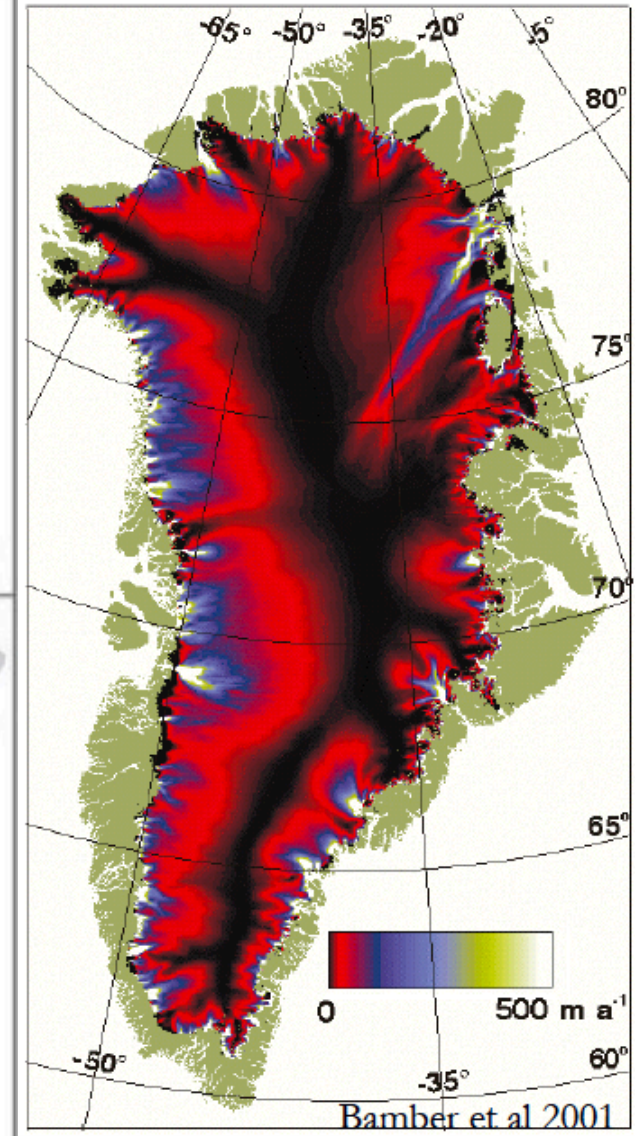
Fault mechanics collaborators:
Nicolas Brantut (*Univ.Col.London*), John D. Platt (*Harvard*),
John W. Rudnicki (*Northwestern*)

The major ice sheets - *not to scale*

Antarctica

Greenland

A

Slessor Glacier

Filchner-Ronne Ice Shelf

Bailey ice stream

Institute ice stream

Evans ice stream

Pine Island Glacier

Byrd Glacier

Ross Ice Shelf

0   >250 m a⁻¹

Bamber et al 2000

0   500 m a⁻¹

Bamber et al 2001

From USGS Fact Sheet fs002-00: Sea Level and Climate
(R. Z. Poore, R. S. Williams, Jr., and C. Tracey)

Average sea level rises due to melting:

    Greenland Ice Sheet: ~ 6.5 m

    West Antarctic Ice Sheet: ~ 8 m

"The West Antarctic ice sheet is especially vulnerable, because much of it is grounded below sea level. Small changes in global sea level or a rise in ocean temperatures could cause a breakup of the two buttressing ice shelves (Ronne/Filchner and Ross)."

Source locations of **glacial earthquakes** (G. Ekström)
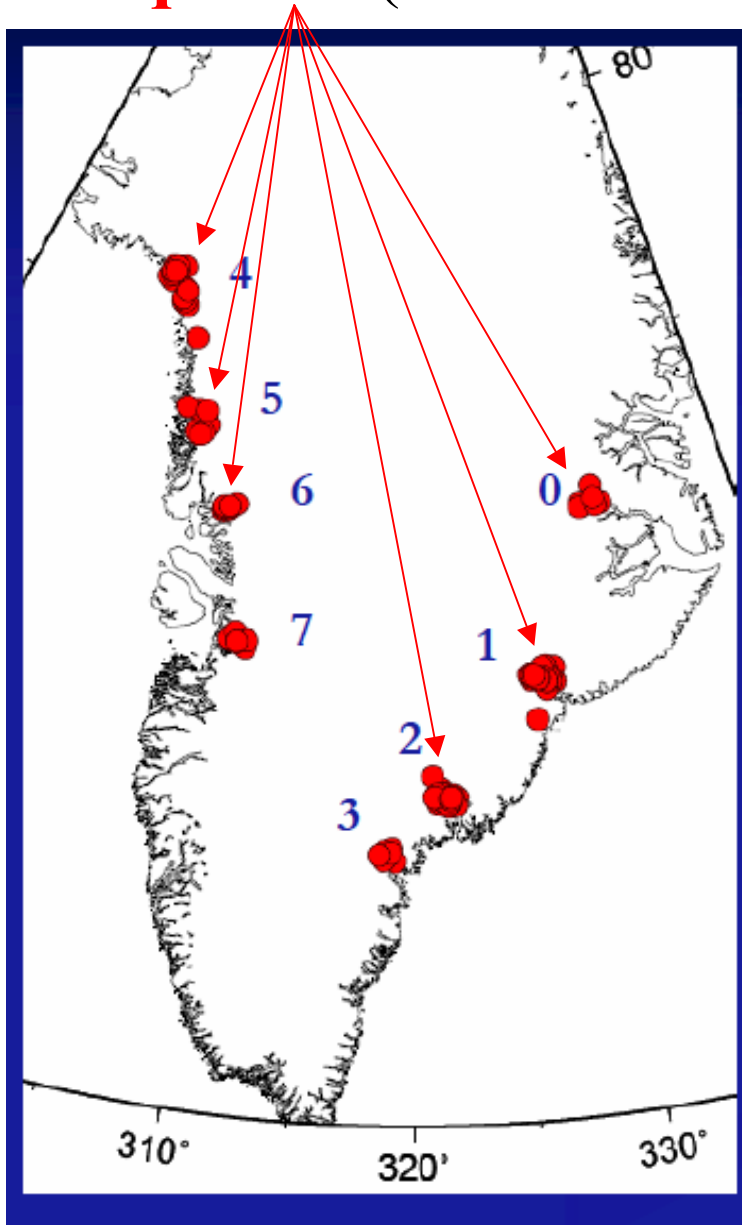

Unusual earthquakes:

• Magnitude $M_{sw} \sim 4.6$ to 5.1, measured at 35-150 sec periods; significant energy in periods between 20 and 100 sec (much longer than for standard earthquakes of similar M_S).

• Distant seismic wave patterns consistent with applying a horizontal point impulse $+\,I$ followed, after ~tens of seconds to minute, by $-\,I$ at shallow source location.

Source locations of **glacial earthquakes** (G. Ekstrom)
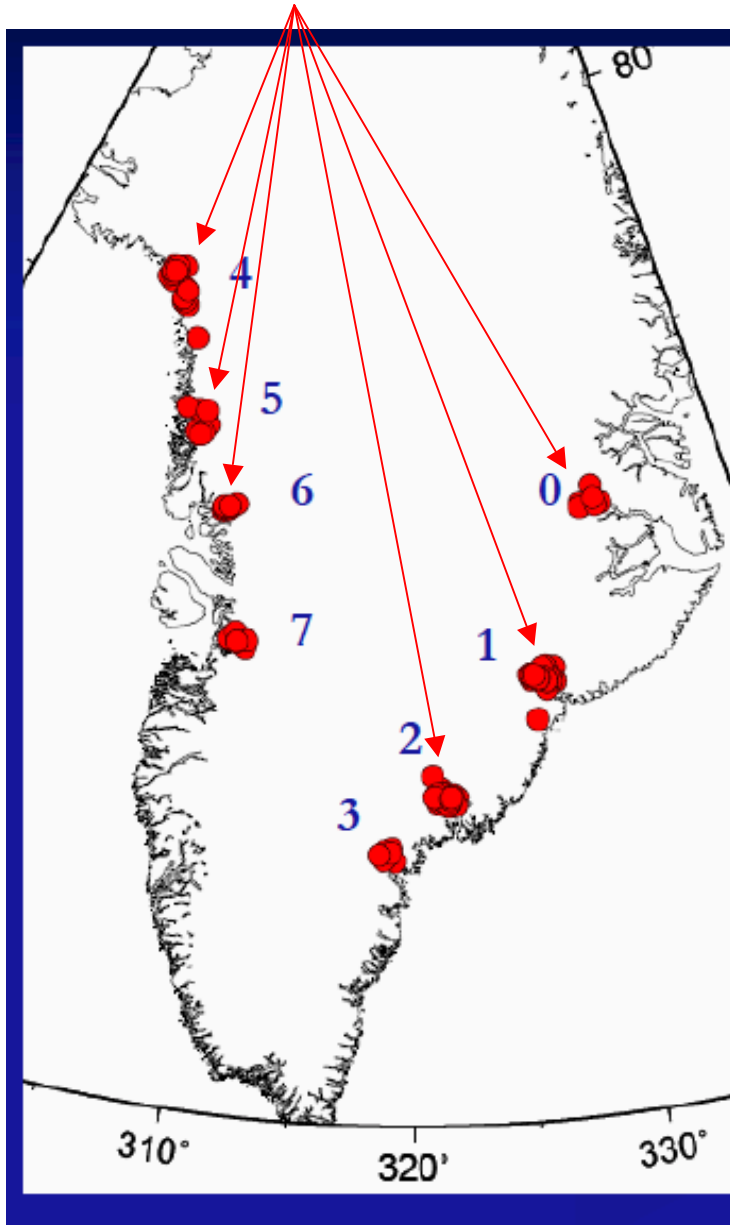
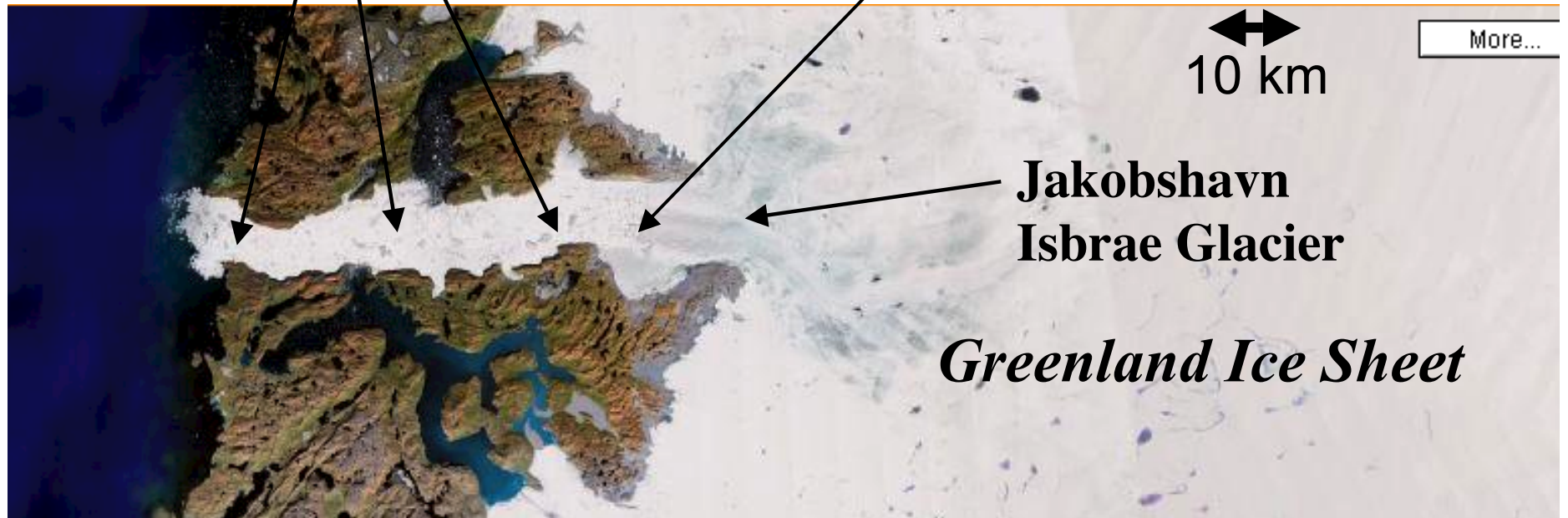Correlation with areas of high ice flow rates -- at major fjords

Bamber et al., 2001
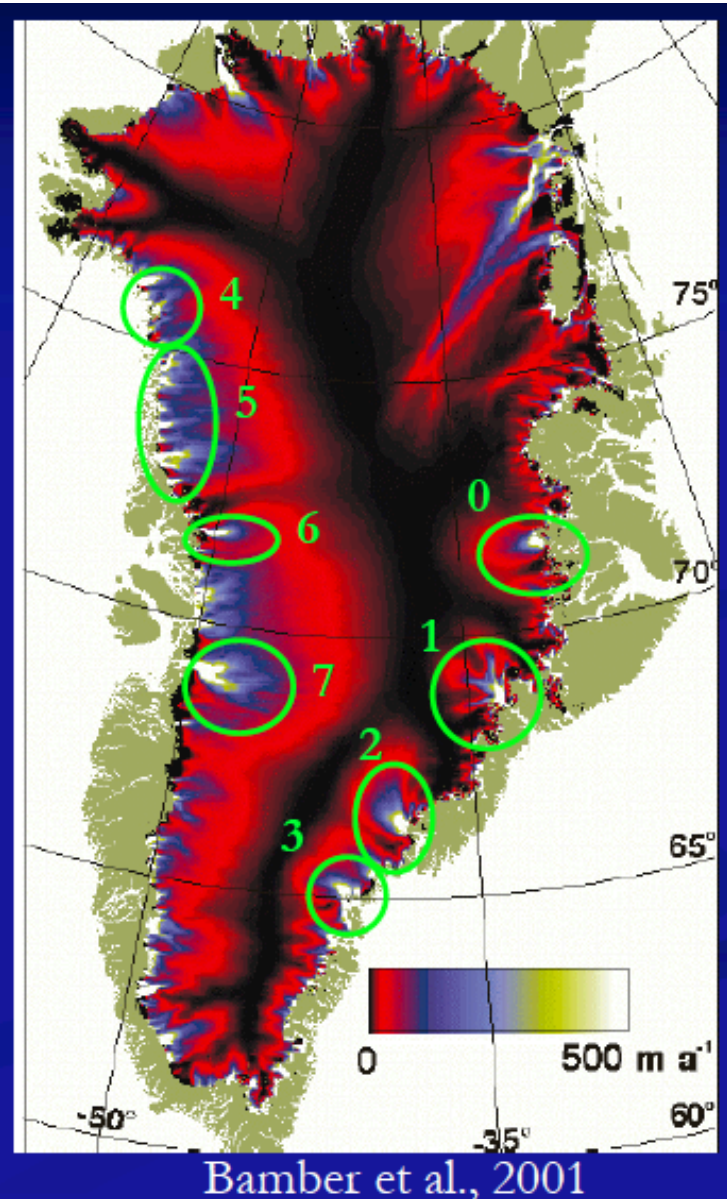
Melange of calved icebergs   Iceberg calving front, glacier terminus

Jakobshavn
Isbrae Glacier
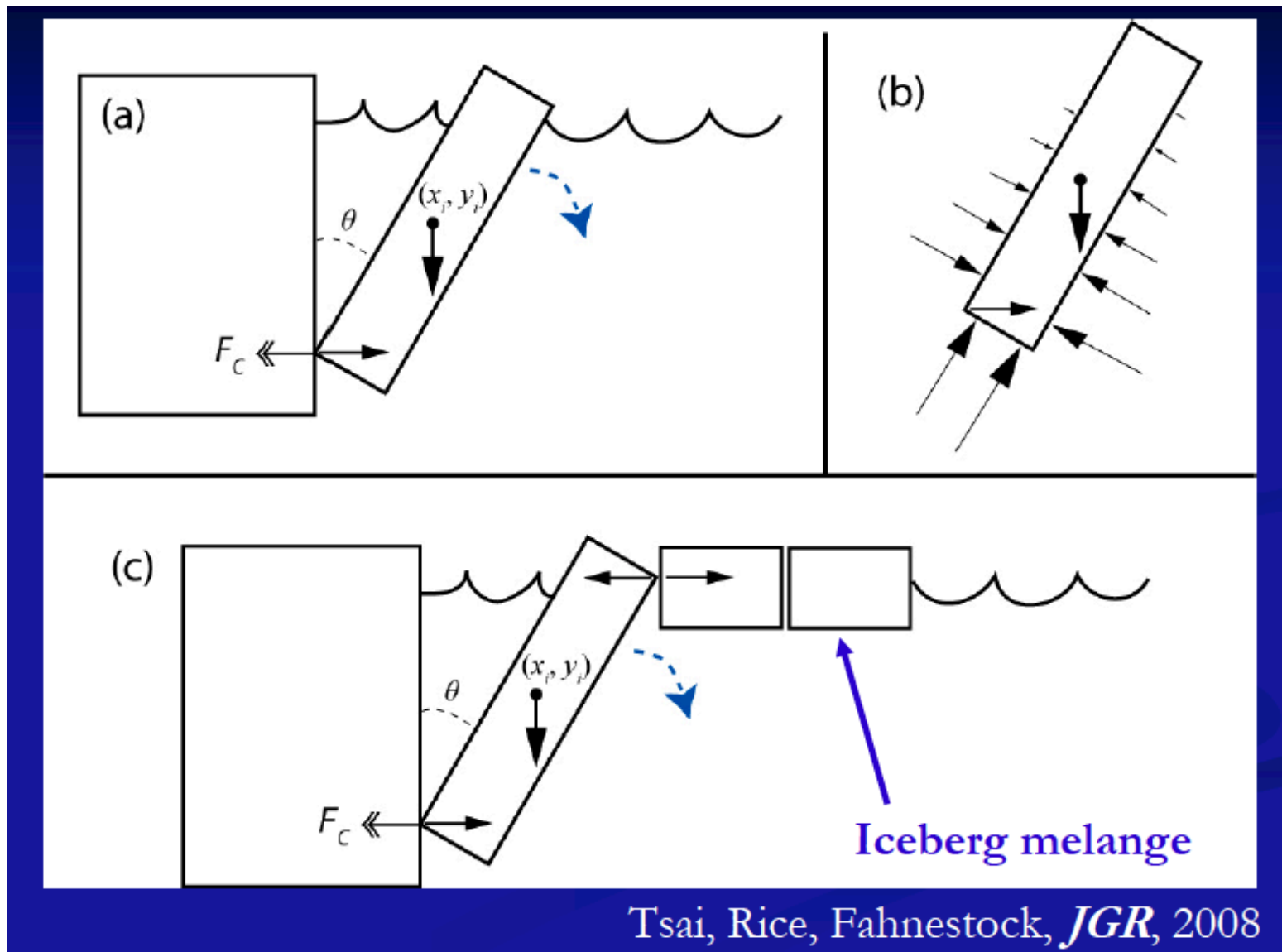
Greenland Ice Sheet

10 km

More...

~10 km

What causes glacial EQs?
- Fast sliding at bed of ice sheet? -- analogous to normal EQs.
- Simple iceberg calving models work best! -- timescale.

(a)

(b)

(c)

Iceberg melange

Tsai, Rice, Fahnestock, *JGR*, 2008
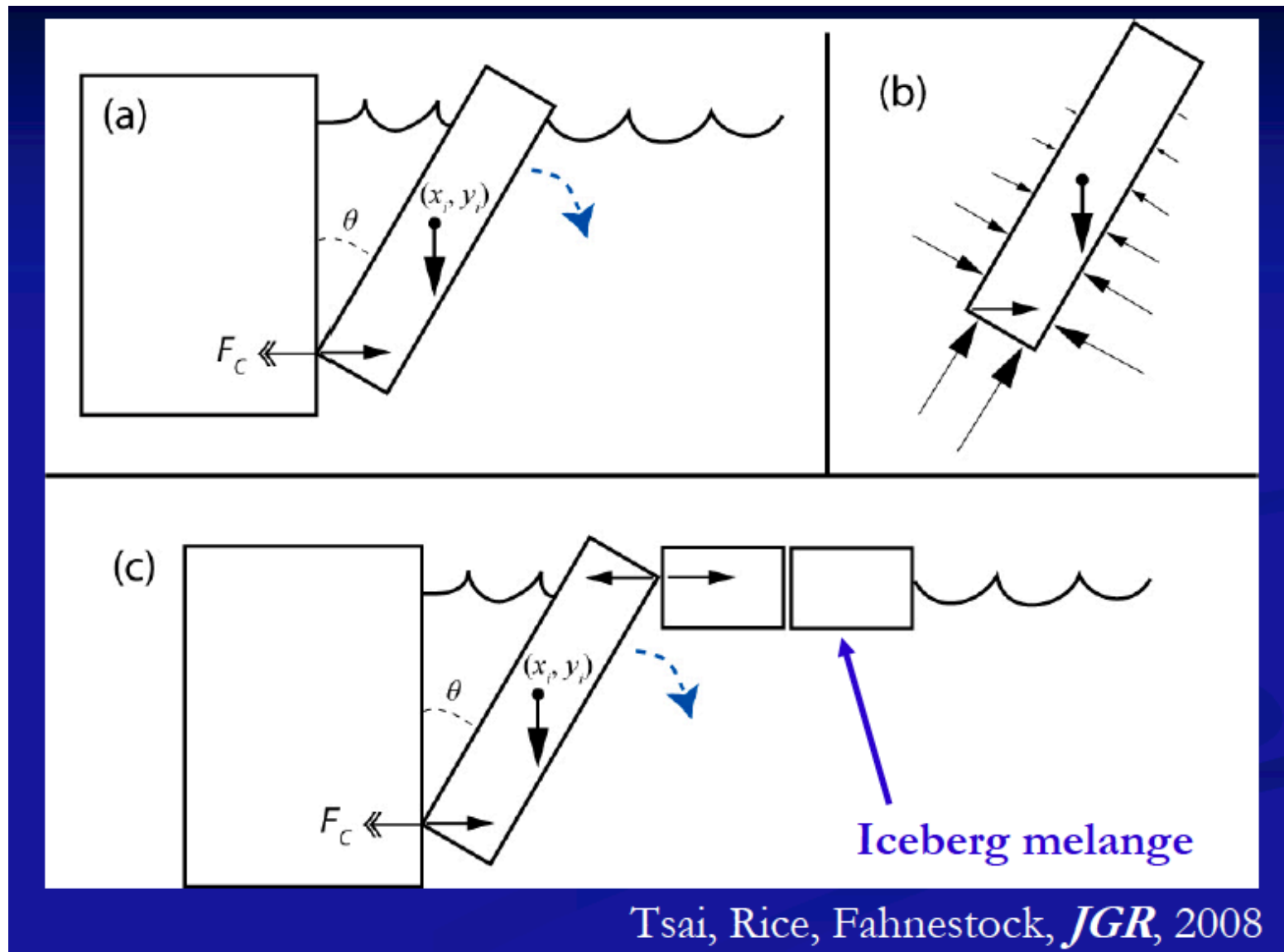
Long period of calved block turnover, because:
1. Long pendulum period (size scale large); 2. Small gravity drive, $\rho_{water} \simeq \rho_{ice}$; 3. Mélange adds effective mass.
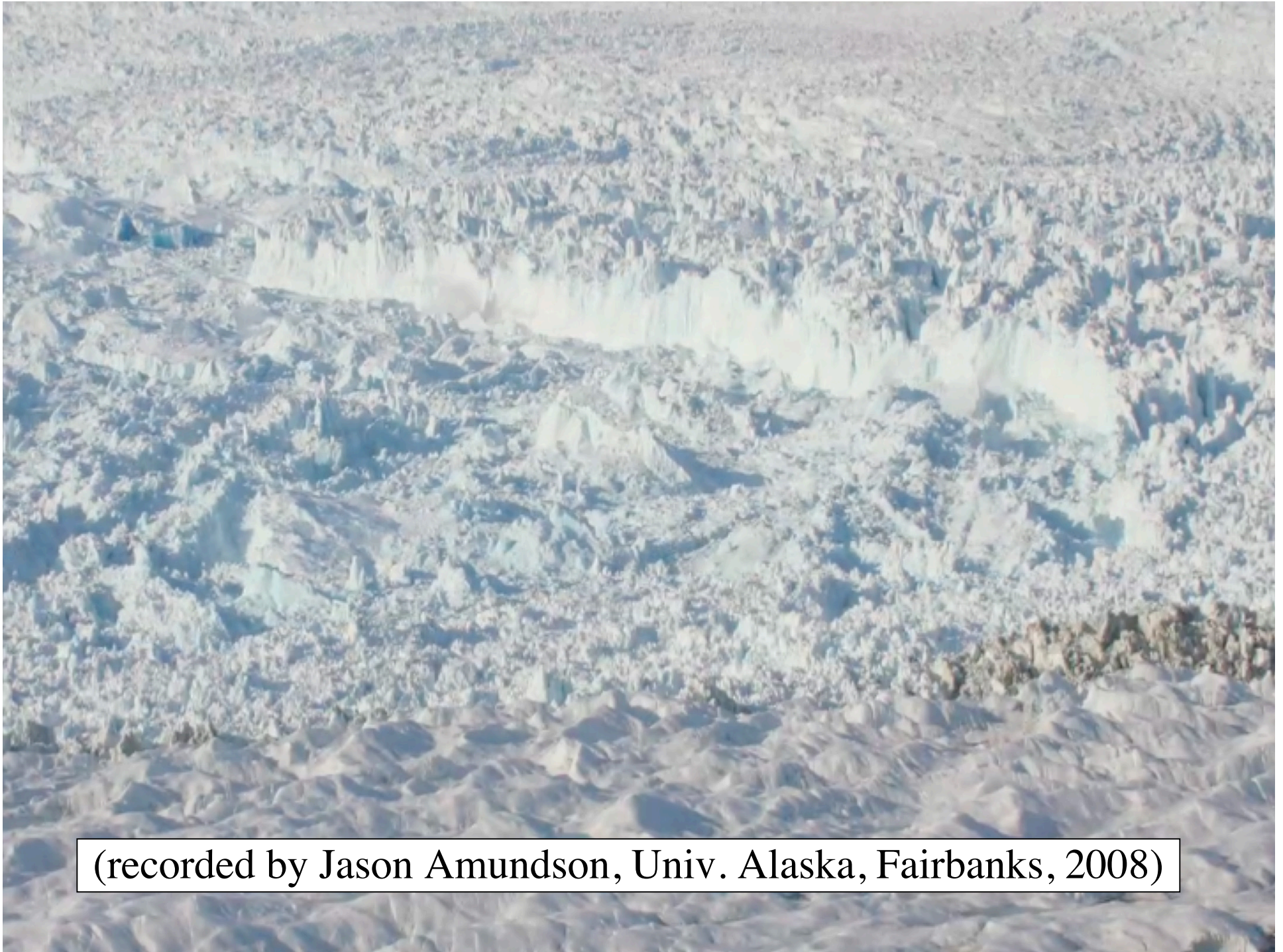
(a)

θ

(x_i, y_i)

F_c «

(b)

(c)

θ

(x_i, y_i)

F_c «

Iceberg melange

Tsai, Rice, Fahnestock, *JGR*, 2008

(recorded by Jason Amundson, Univ. Alaska, Fairbanks, 2008)

(recorded by Jason Amundson, Univ. Alaska, Fairbanks, 2008)

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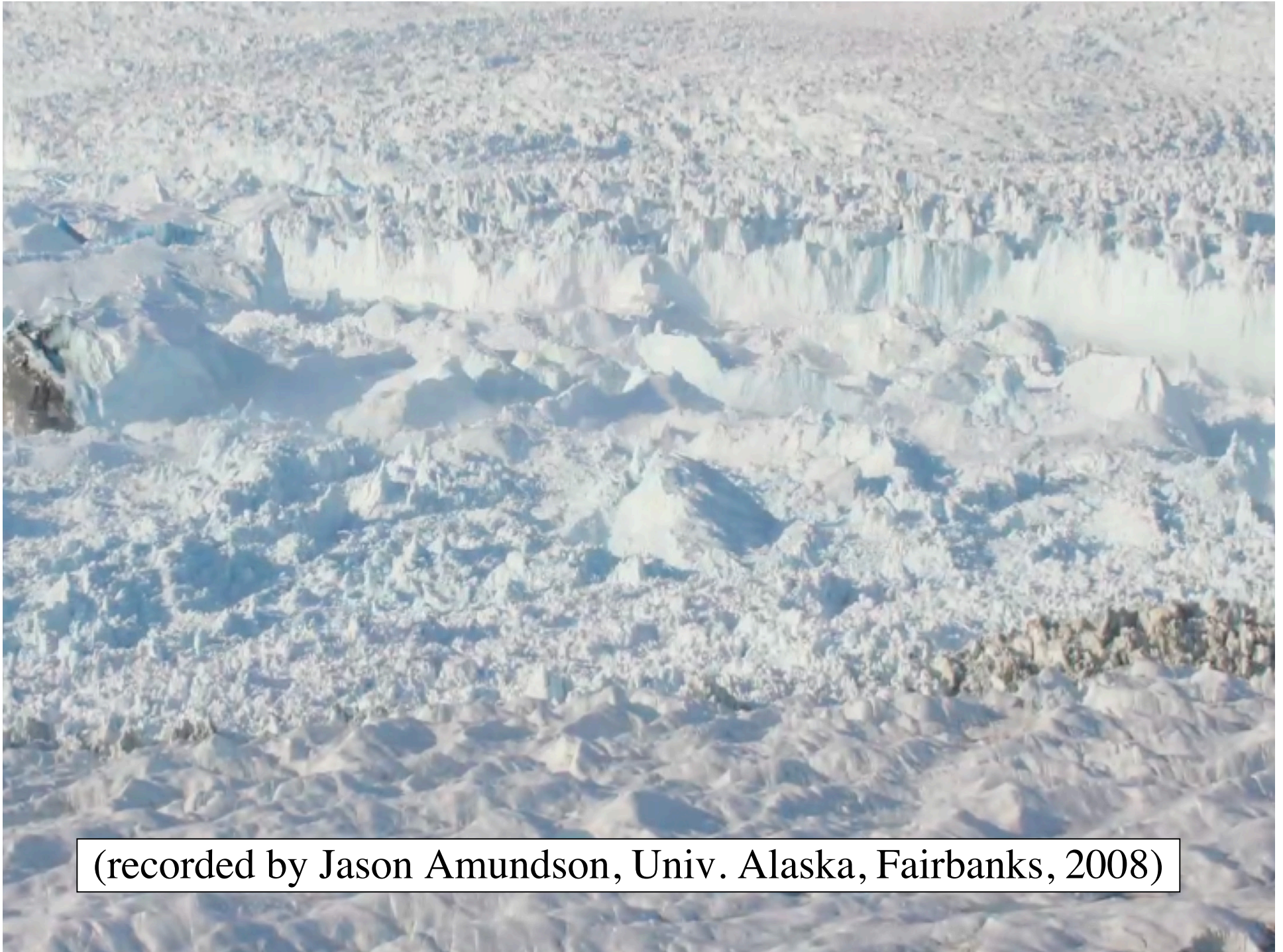
(recorded by Jason Amundson, Univ. Alaska, Fairbanks, 2008)

(recorded by Jason Amundson, Univ. Alaska, Fairbanks, 2008)
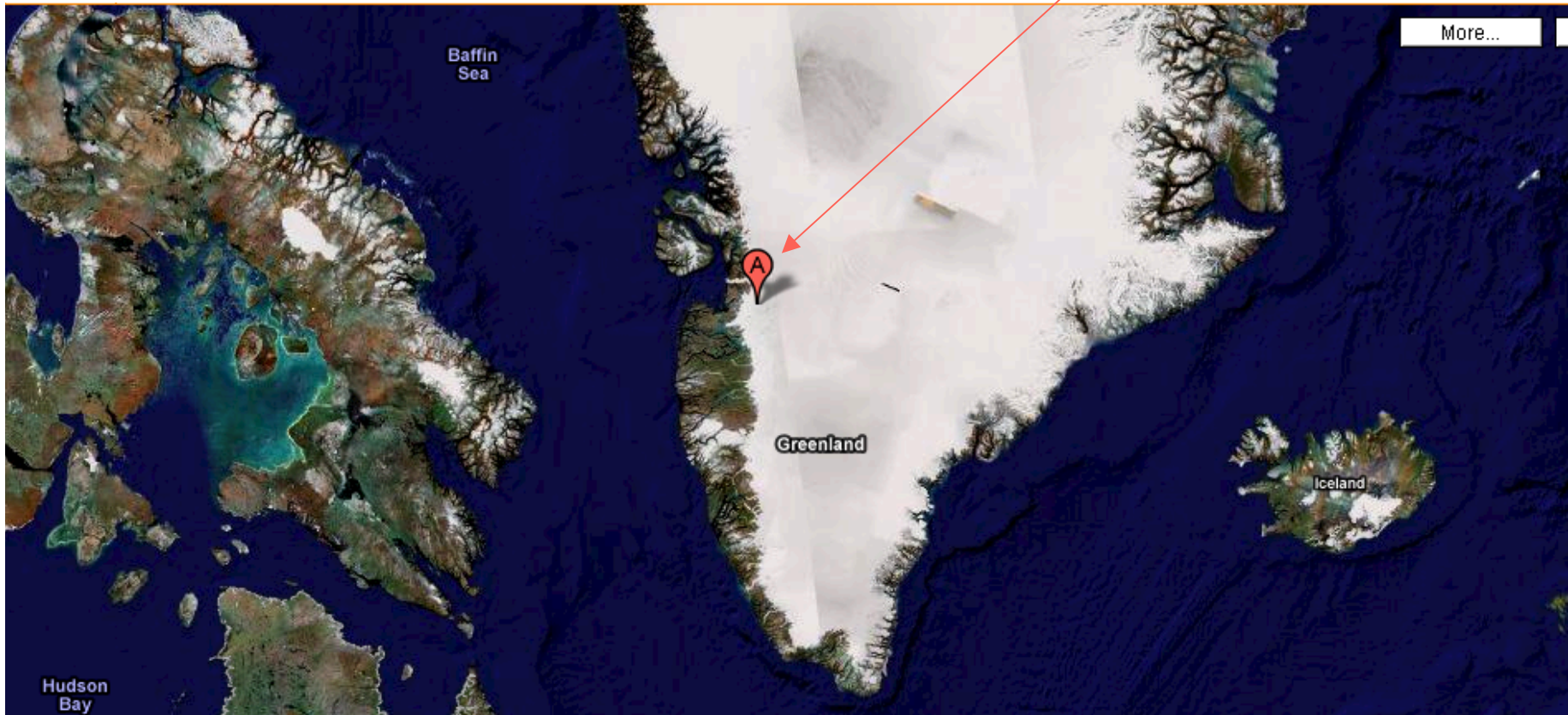
(recorded by Jason Amundson, Univ. Alaska, Fairbanks, 2008)

A natural hydraulic fracture of interest for evaluating scenarios of accelerated deglaciation

Where are we?
A below

Study motivated by the paper *Fracture Propagation to the Base of the Greenland Ice Sheet During Supraglacial Lake Drainage*, by Das, Joughin, Behn, Howat, King, Lizarralde & Bhatia, *Science*, May 2008.

Melange of calved icebergs   **Iceberg calving front, at glacier terminus**

Jakobshavn Isbrae Glacier

Greenland Ice Sheet

A

10 km
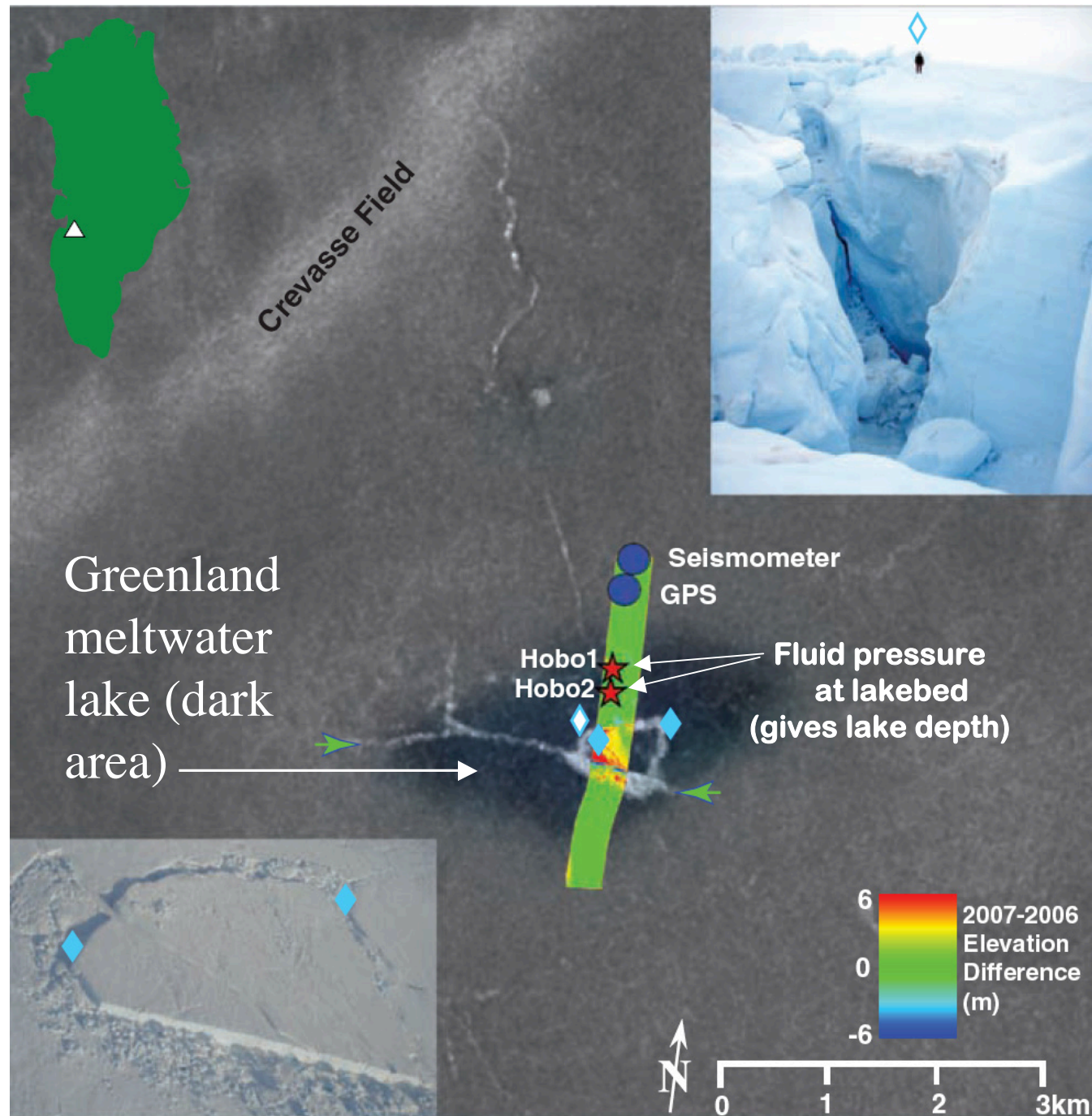
©2008 Google - Imagery ©2008 TerraMetrics

Study motivated by the paper *Fracture Propagation to the Base of the Greenland Ice Sheet During Supraglacial Lake Drainage*, by Das, Joughin, Behn, Howat, King, Lizarralde & Bhatia, *Science*, May 2008.

(Das et al., *Sci.,* 2008)

Early October 2006 SAR image (gray-scale background) overlaid with a semi-transparent image recorded by NASA's Moderate Resolution Imaging Spectroradiometer (MODIS) showing the lake extent (blue) on 29 July 2006.

Crevasse Field

Greenland meltwater lake (dark area)

Seismometer
GPS

Hobo1
Hobo2

Fluid pressure at lakebed (gives lake depth)

6
0
-6

2007-2006 Elevation Difference (m)

N

0   1   2   3km

- *Supraglacial meltwater lake* began filling July 2006
- Maximum ~0:00 29 July 2006, $44 \times 10^6 \, m^3$, 5.6 km^2
- Level slowly/steadily falls, 15 mm/hr
- Rapid from 16:00-17:30, max 12 m/hr ($Q > 10,000 \, m^3/s$), avg $Q \sim 8,700 \, m^3/s$ [Compare, Niagra Falls $Q \sim 6,000 \, m^3/s$]

(Das et al., *Sci.*, 2008)

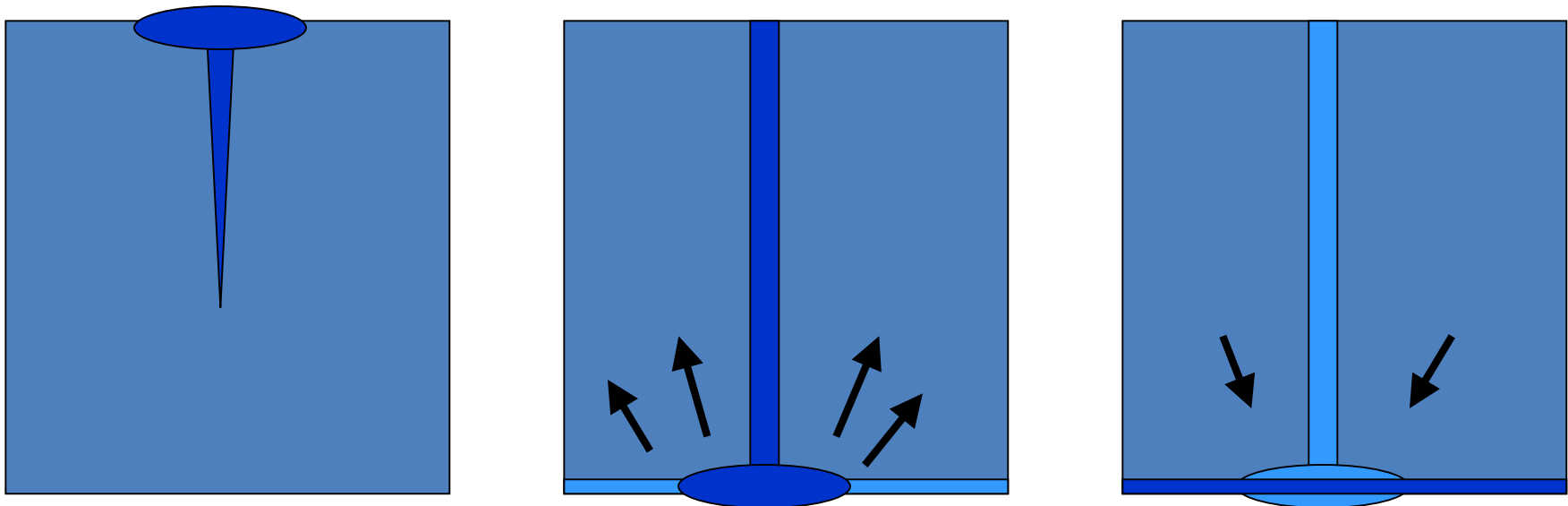Falling lake level [m] (right scale)

Ice surface uplift [m] (left scale)
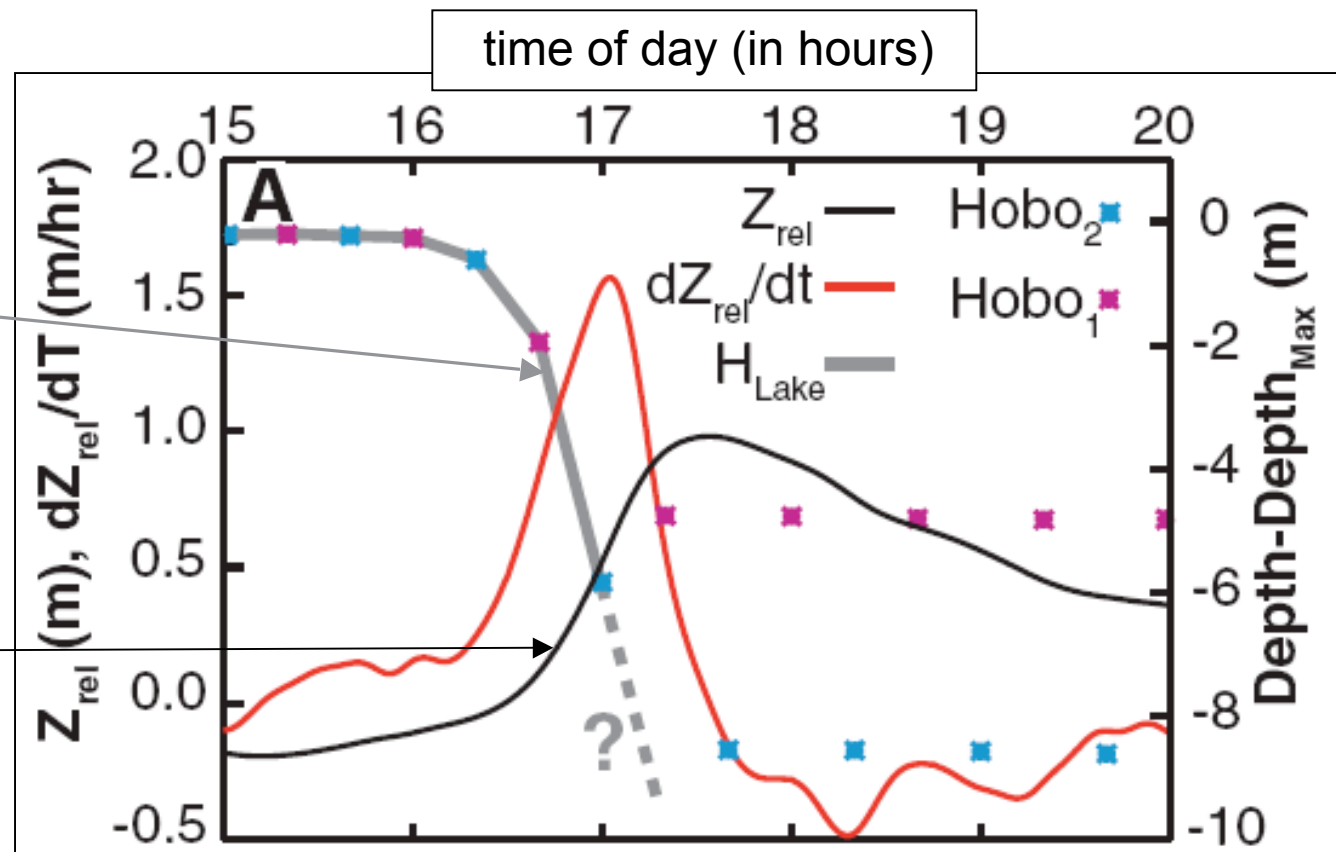
# *Interpretation*

- Initially: Crack/moulin system gradually propagates to bed by *Weertman* gravitational instability, $\rho_{water} > \rho_{ice}$.

- Middle Stage: Hydraulic cracking and flooding along bed by over-pressure, $p > \sigma_o$ (σ_o = ice overburden pressure).

- End: Fracture closes, subglacial water layer drains.

Rubin, **Propagation of magma-filled cracks** [Annu. Rev. Earth Planet. Sci., 1995]

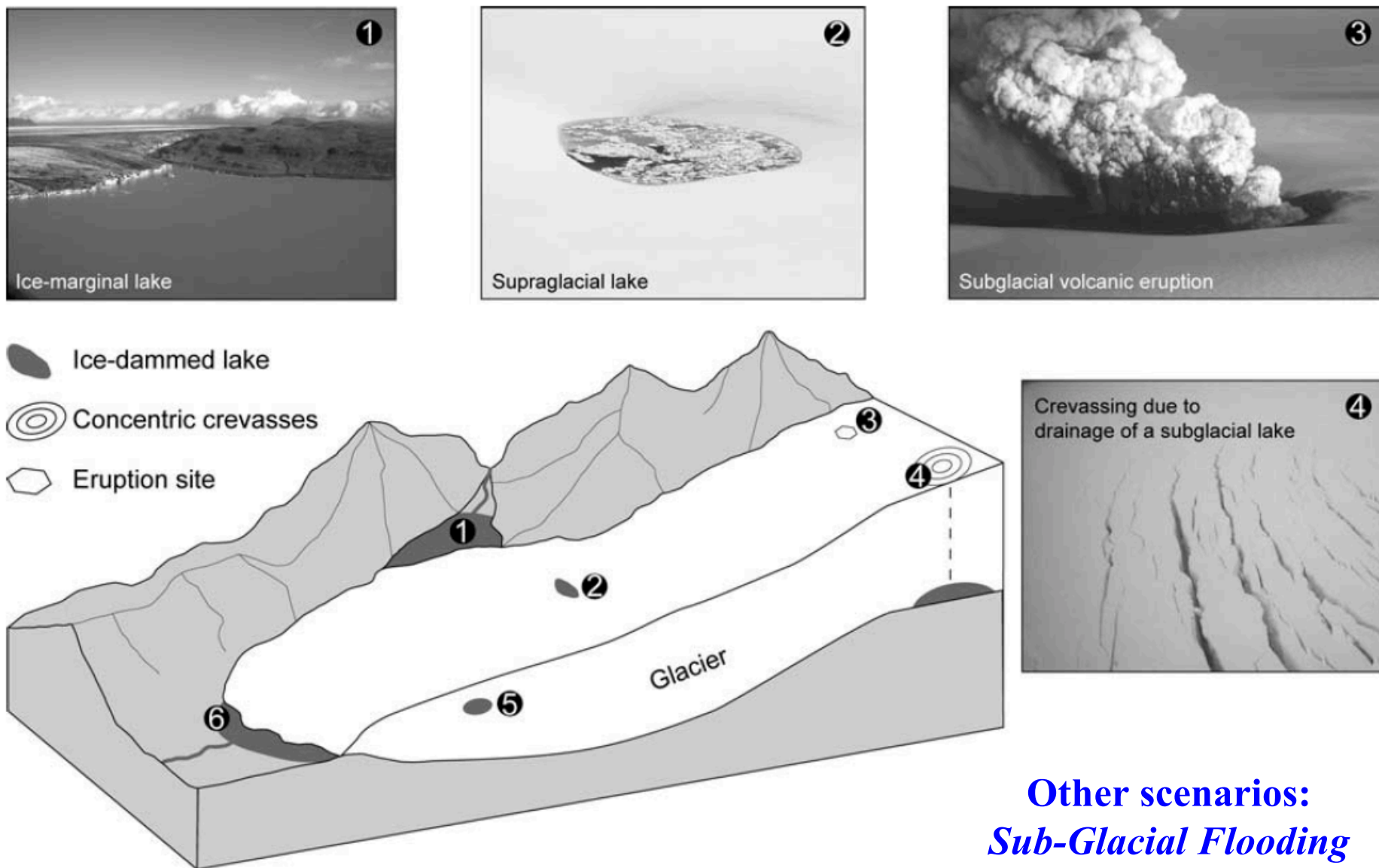$$\rho_{magma} < \rho_{rock}$$
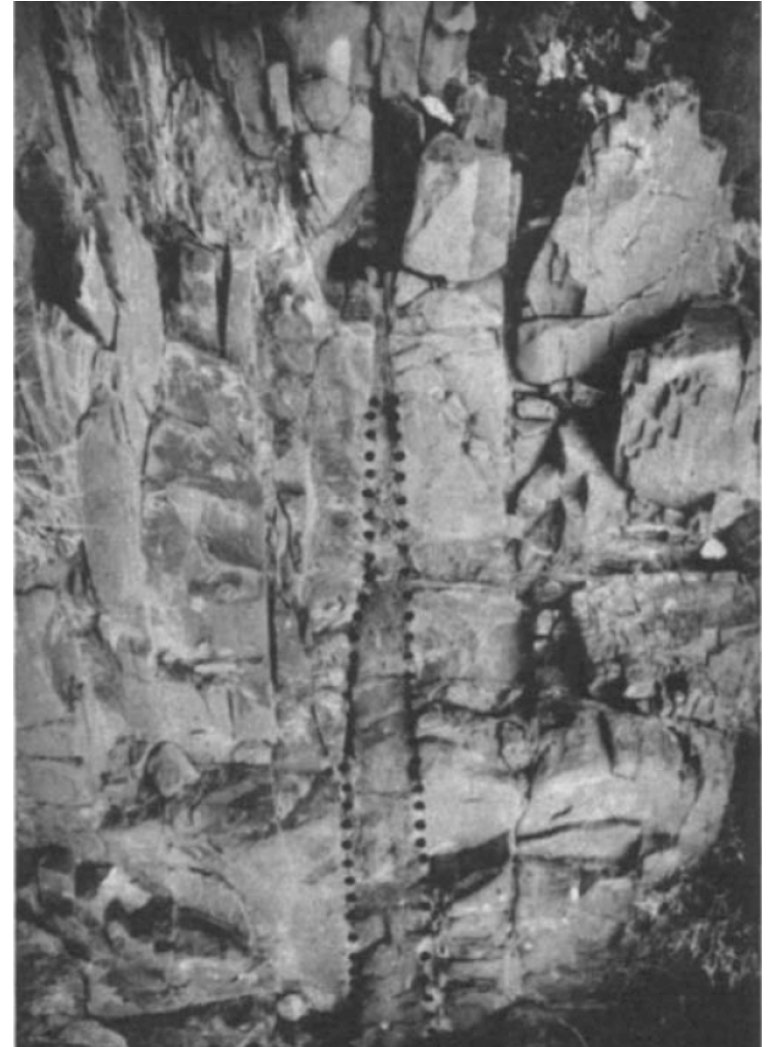
Basaltic dike at tip of Reykjanes Peninsula, southwest Iceland, exposed by glacial erosion (did not make it to surface). Thickness = 40 cm.

Dike (boundaries dotted) terminating in shear zone on Colorado Plateau.

1 Ice-marginal lake

2 Supraglacial lake

3 Subglacial volcanic eruption

4 Crevassing due to drainage of a subglacial lake

Ice-dammed lake
Concentric crevasses
Eruption site

Glacier

Other scenarios:
Sub-Glacial Flooding (Jökulhlaup)

Figure 1. Reservoir sites and meltwater sources for jökulhlaups.

from: Roberts, M. J. (2005), Jökulhlaups: A reassessment of floodwater flow through glaciers, *Rev. Geophys.,* 43, RG1002.

Alaska Looks for Answers in Glacier's Summer Flood Surges

New York Times July 22, 2013

First observed in July 2011, also in July 2012 & 2013.

"… in July 2011, … an estimated ten billion gallons gushed out in three days … two smaller bursts this year …"

Mathew Ryan Williams for The New York Times

Visitors leaving caves under the Mendenhall Glacier, near Juneau, Alaska. Unpredictable flood surges have elevated concerns.
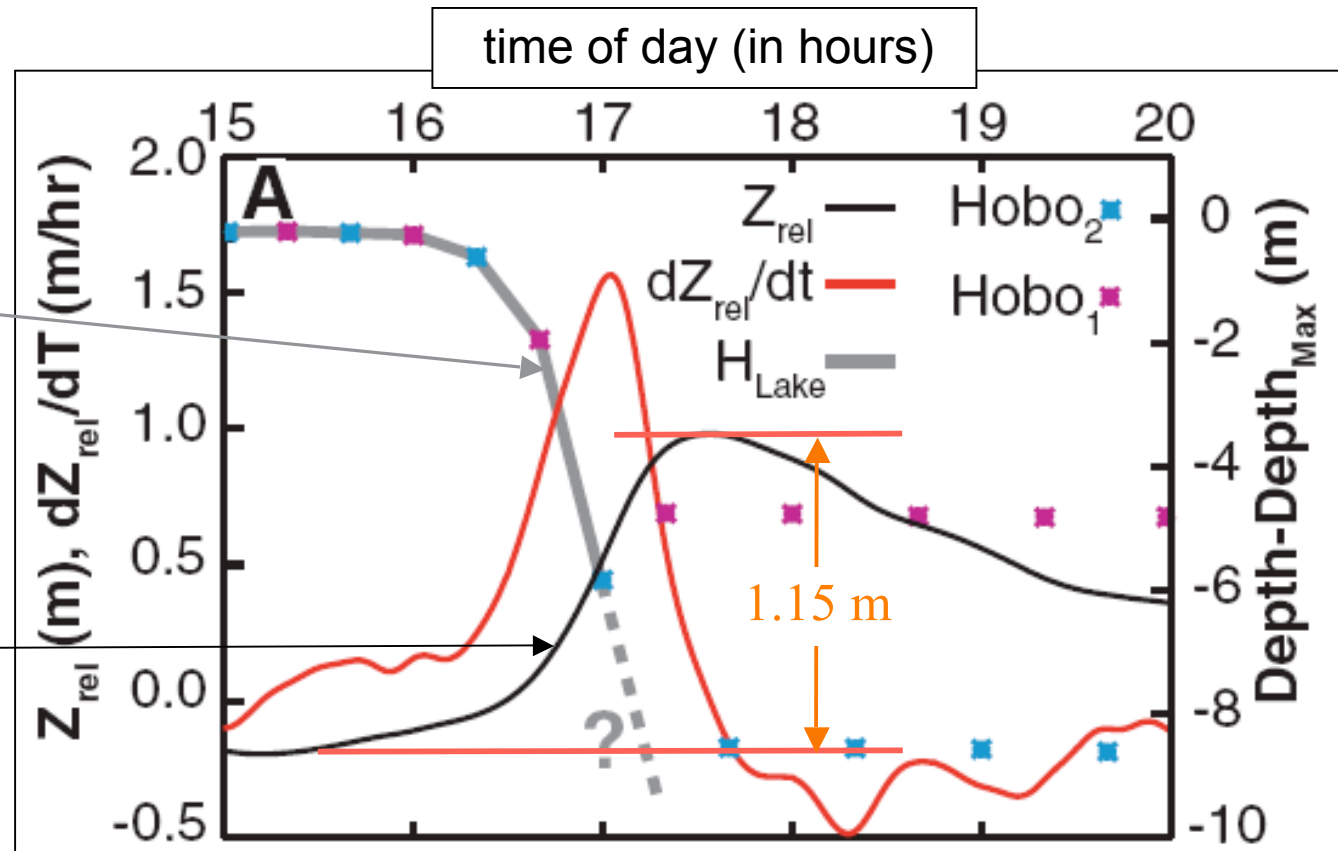
By KIRK JOHNSON

JUNEAU, Alaska -- …. unpredictable flood surges at the Mendenhall Glacier, about 14 miles from downtown Juneau, Alaska's capital ….

(Das et al., *Sci.*, 2008)

Falling lake level [m] (right scale)

Ice surface uplift [m] (left scale)

Approximate radius R of sub-glacial fracture at full lake discharge :

$$\pi R^2 \times \text{Uplift of } 1.15 \text{ m} = \text{Lake volume of } 44 \times 10^6 \text{ m}^3 \Rightarrow R \approx 3.5 \text{ km}$$

$$\Rightarrow \quad \textbf{Average growth speed} \approx R \,/\, 1.2 \text{ hr} \approx 3 \text{ km} \,/\, \text{hr}$$

$$\Rightarrow \textbf{Reynolds number Re} \text{ for flow in fracture} \approx \frac{3 \text{ km} \,/\, \text{hr} \times 0.5 \text{ m}}{10^{-6} \text{ m}^2/\text{s}} \approx 4 \times 10^5$$

Gioia & Chakraborty [*PRL*, 2006] replot, Nikuradse [1933] rough-wall pipe-flow data
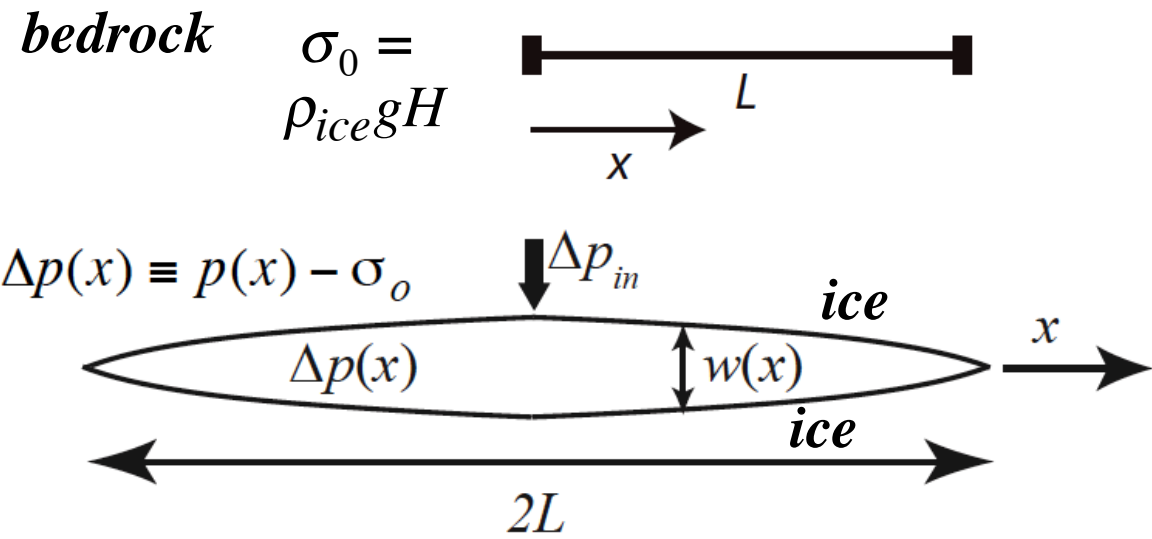
$$f = \frac{8\tau_{wall}}{\rho U^2}$$

$$\frac{R}{r} = \frac{\text{pipe radius}}{\text{roughness ampl.}} \approx \text{corresponds to } \frac{h}{k}$$

← Fracture opening width *h*.

Roughness *k* is typically a few mm to a few cm, from G. K. C. Clarke [*J. Glac.*, 2003] modeling of glacial outburst flooding. (Clarke reports $n_{Manning}$.)
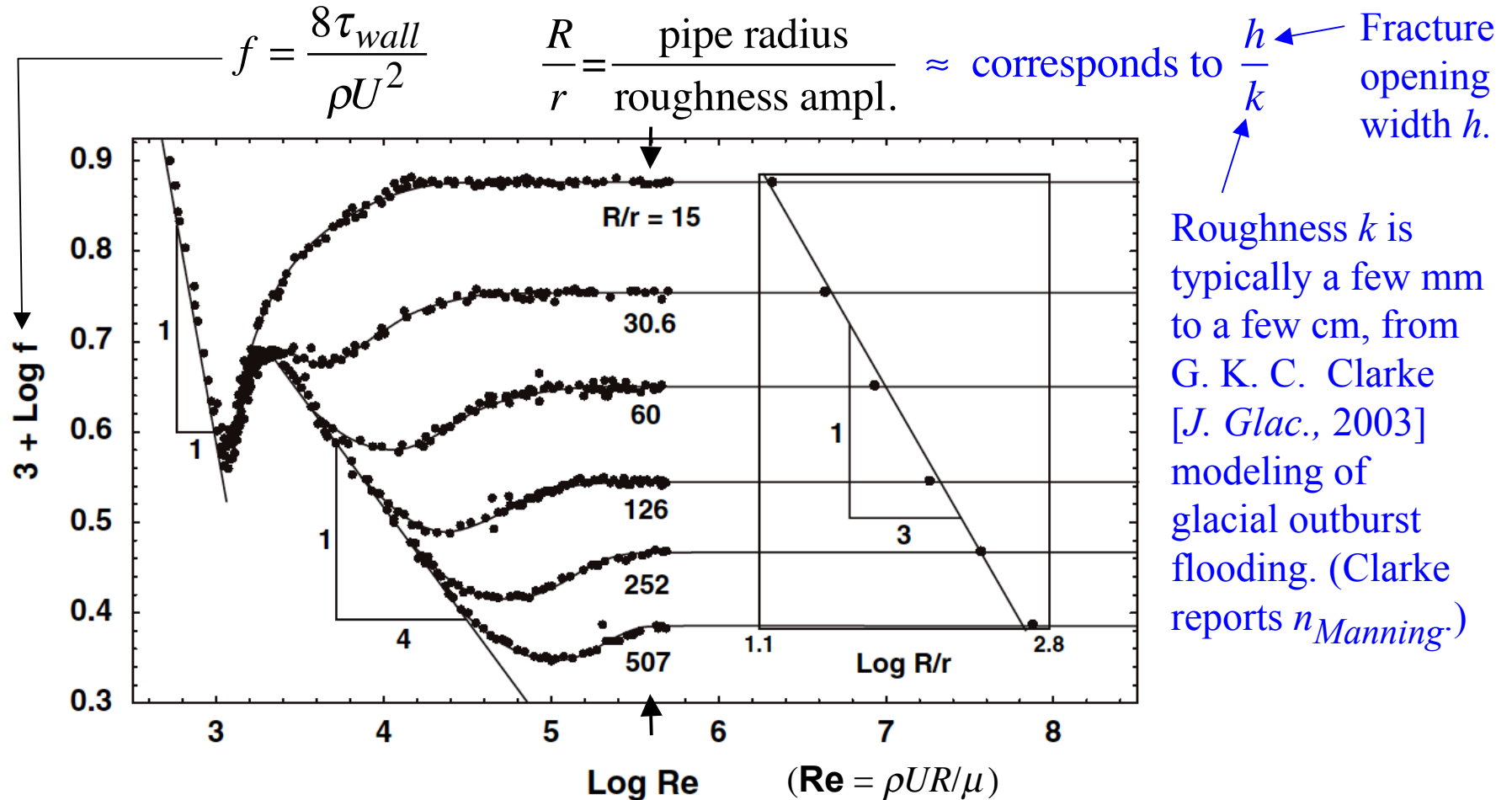
Nikuradse's data, Darcy-Weisbach *f* versus Reynolds number Re, pipe flow with rough walls. At large Re, *f* becomes independent of Re.

Inset: Manning-Strickler scaling, $f \approx 0.143(r/R)^{1/3}$. Means $f \approx 0.143(k/h)^{1/3}$.

GPS Station

x_0

Vertical Conduit

$2u(0)$

Linear Elastic Ice

Pressure Gradient Driving Flow

σ_0

Turbulent Flow of Water Driving Basal Fracture

H

$h(x)$

z

ice

dp/dx

$U(x)$

U_{tip}

bedrock

$\sigma_0 = \rho_{ice}gH$

L

x

$\Delta p(x) \equiv p(x) - \sigma_o$

Δp_{in}

ice

x

$\Delta p(x)$

$w(x)$

ice

$2L$
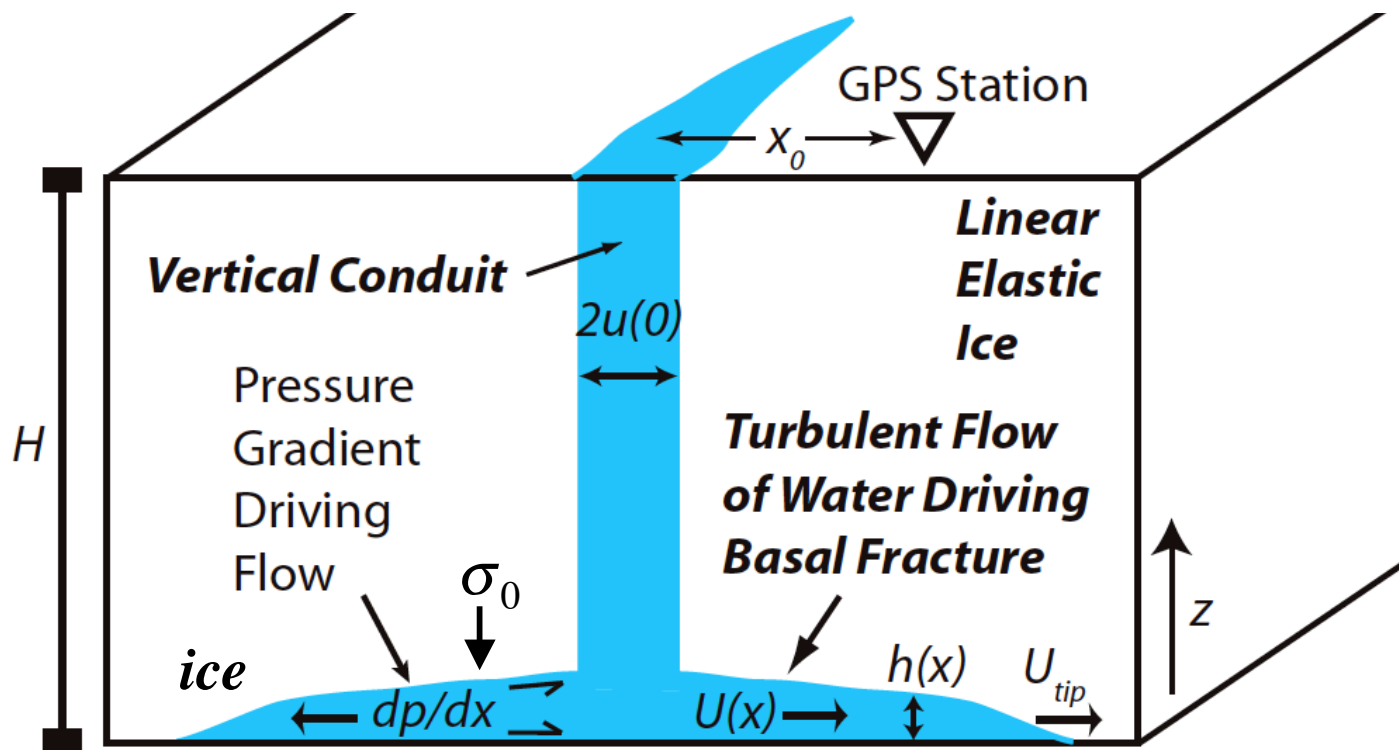
$h(x,t) \approx \xi w(x,t), \quad \xi \approx 0.55$

Governing Equations, valid for range $L << H$ = ice thickness $\approx$ 1 km

$$p(x,t) - \sigma_0, \quad h(x,t) = \xi w(x,t), \quad \text{and} \quad U(x,t) \text{ related by}$$

- Linear elasticity for cracks:

  (w = opening in homog. ice; h = opening in ice/rock)

$$p(x,t) - \sigma_0 = \frac{E'}{2\pi\xi} \int_{-L}^{L} \frac{\partial h(x',t)}{\partial x'} \frac{dx'}{x - x'}$$

- Fracture mechanics, ice-rock interface:

  (toughness $K_{Ic} \approx 0.1$ MPa m$^{1/2}$; negligible, for $L > \sim 10$ m)

$$K_{Ic} = 0 \quad \Rightarrow$$
$$r^{1/2}\sigma_{ij} \to 0 \text{ as } r \to 0$$

- Conserv. of fluid mass (fluid volume):

$$\frac{\partial(hU)}{\partial x} + \frac{\partial h}{\partial t} = 0$$

- Manning-Strickler turbulent flow:

  (k = Nikuradse roughness scale)

$$-h\frac{\partial p}{\partial x} = \underbrace{0.0357\rho U^2 \frac{k^{1/3}}{h^{1/3}}}_{2\tau_{wall}}$$

Self-Similar Solution (2D plane-strain, $L << H$ [Tsai & Rice, *JGR, 2010*])
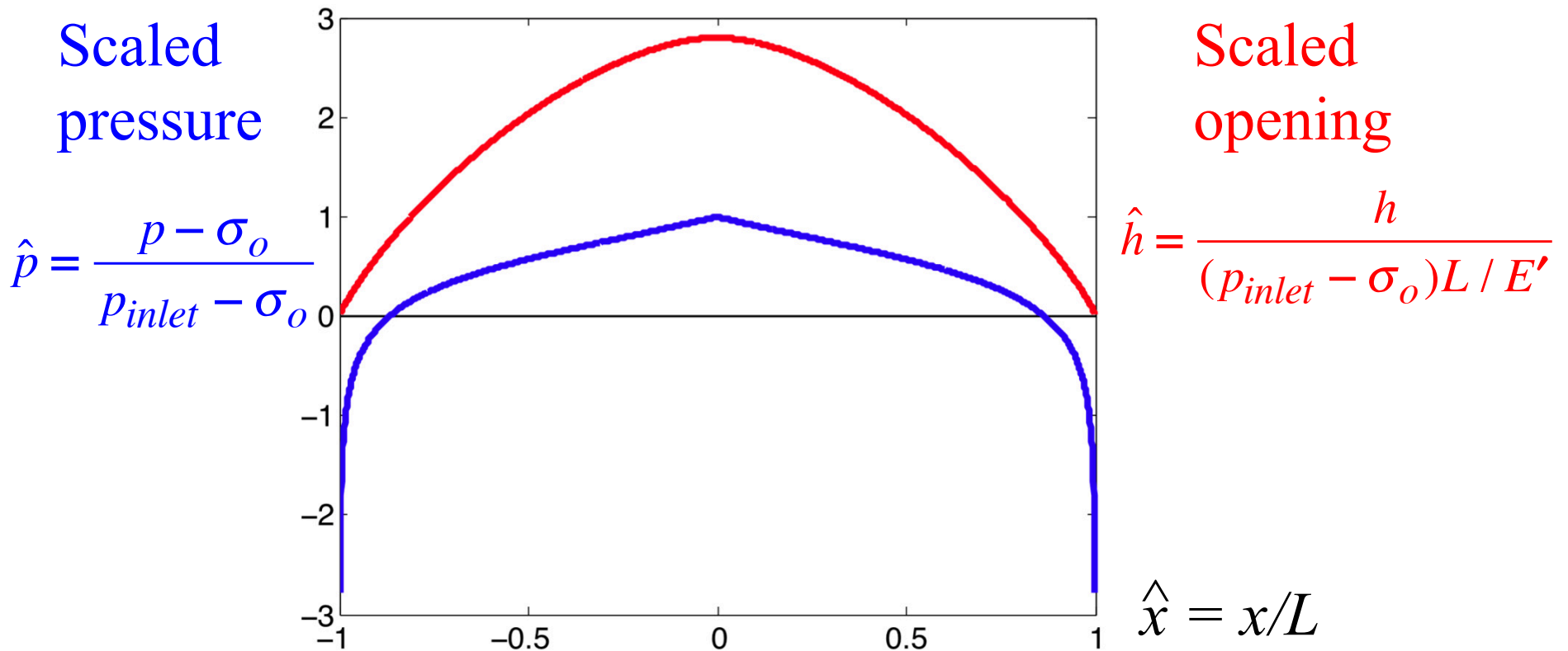
(Approach similar to Adachi and Detournay [*Int. J. Numer. Anal. Meth. Geomech.*, 2002], who solved the same problem for a power-law viscous fluid in locally laminar flow.)

$p_{inlet} - \sigma_o$
(constant, for solution shown)

$h(x,t)$

$x \rightarrow$

$U(x,t)$

r

θ

$Q \quad Q$

$L(t)$
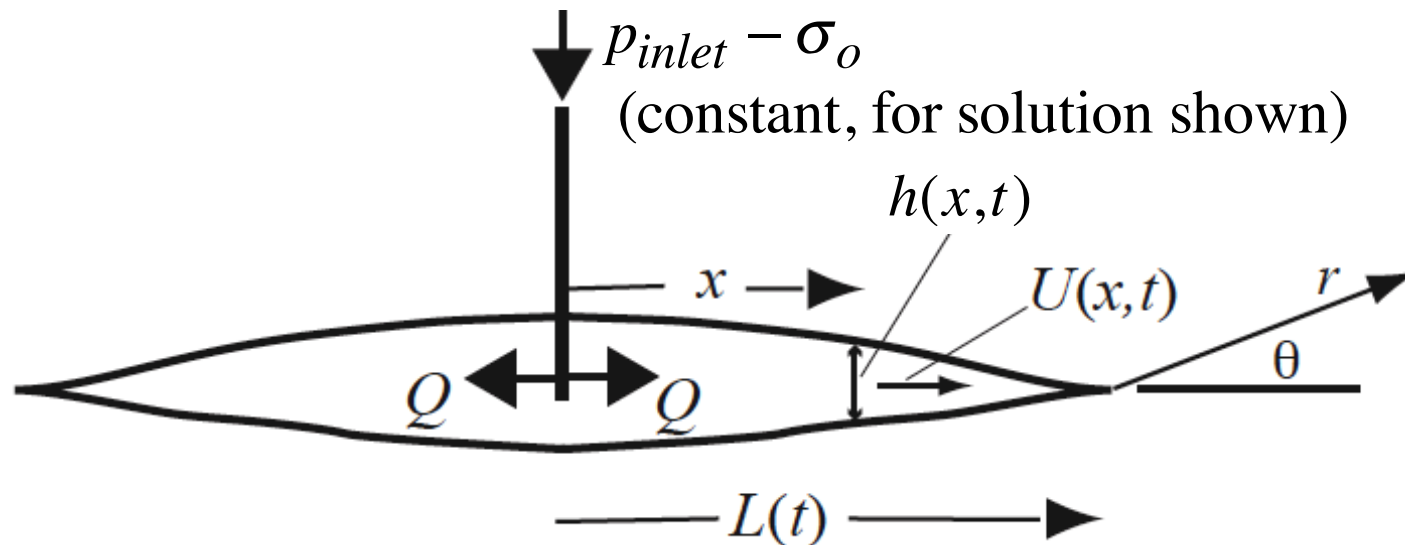
Our case (turbulent, high **Re**): $\tau_{wall} = \dfrac{f}{8}\rho U^2 = -\dfrac{1}{2}h\dfrac{\partial p}{\partial x}$, $f = 0.143\left(\dfrac{k}{h}\right)^{1/3}$

$$L(t) = C\,t^{6/5}, \quad h(x,t) \sim C\,t^{6/5}\,F(x/L(t)),$$

$$p(x,t) - p_o = G(x/L(t)), \quad U(x,t) = C\,t^{1/5}\,H(x/L(t)).$$

Scaled pressure

$$\hat{p} = \frac{p - \sigma_o}{p_{inlet} - \sigma_o}$$

Scaled opening

$$\hat{h} = \frac{h}{(p_{inlet} - \sigma_o)L / E'}$$

$$\hat{x} = x/L$$

- Crack growth rate:

$$\frac{dL(t)}{dt} = 5.17 \sqrt{\frac{p_{inlet} - \sigma_o}{\rho}} \left(\frac{p_{inlet} - \sigma_o}{E'} \right)^{2/3} \left(\frac{L(t)}{k} \right)^{1/6}$$

Making contact with the observations [Das et al., '08] of surface-lake drainage driving hydraulic fracture near a margin of the Greenland Ice Sheet, and using analytical results for self-similar plane strain fracture:

For $p_{hydrostatic} - \sigma_o = 0.87$ MPa , $k = 1$ cm, $L = 1$ km:
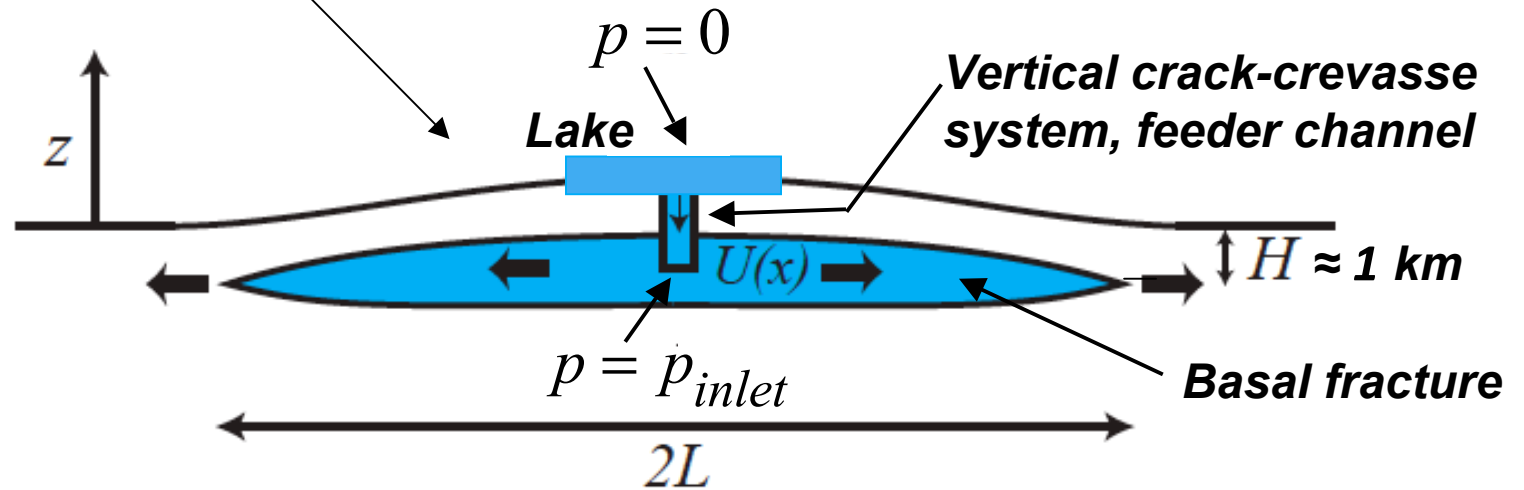
- If $p_{inlet} - \sigma_o = p_{hydrostatic} - \sigma_o$, $U_{tip} = 9.4$ km/hr , $h_{avg} = 0.13$ m.

- If $p_{inlet} - \sigma_o = 0.5\,(p_{hydrostatic} - \sigma_o)$, $U_{tip} = 4.3$ km/hr , $h_{avg} = 0.07$ m.

- **If also k decreased by factor of 5, to $k = 2$ mm, $U_{tip} = 5.6$ km/hr , $h_{avg} = 0.07$ m.**

- Young's modulus $E = 6.2$ GPa at $-5°$C [Jellinek et al., '55] and Poisson's ratio $\nu = 0.3$ [Vaughan, '95], gives **$E' = 6.8$ GPa**.
- Liquid density $\rho = 1000$ kg/m³, ice density $\rho_{ice} = 910$ kg/m³.
- Ice thickness **$H = 980$ m** [Das et al., '08], so $p_{hydrostatic} - \sigma_o = 0.87$ MPa.
- Dependence of U on channel wall roughness k is weak (power law exponent $= 1/6$); estimate $k = 1$ cm, which is consistent with $n_{Manning} \sim 0.018$ s m$^{-1/3}$.

Finite L / H

Features not to scale!

$p = 0$

Lake

Vertical crack-crevasse system, feeder channel

z

$U(x)$

$H \approx 1$ km
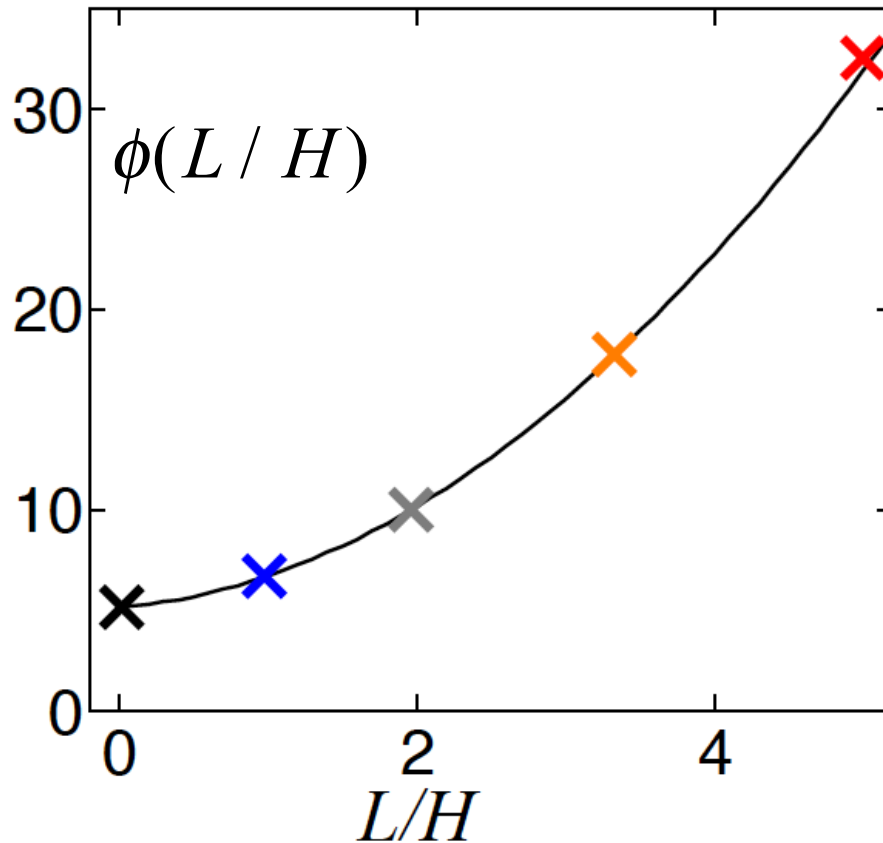
$p = p_{inlet}$

Basal fracture

$2L$

$$p_{hydrostatic} \equiv \rho_{water} g H \; \geq \; p_{inlet} \; \geq \; \rho_{ice} g H \equiv \sigma_o$$

Schematic for turbulent hydraulic fracture

(Analysis simplified by treating ice
and bed as a *homogeneous* medium.)

Crack growth rate =

$$\frac{dL}{dt} \equiv U_{tip} = \left(\frac{p_{inlet} - \sigma_o}{\rho}\right)^{1/2} \left(\frac{p_{inlet} - \sigma_o}{E'}\right)^{2/3} \left(\frac{L}{k}\right)^{1/6} \phi(L/H)$$
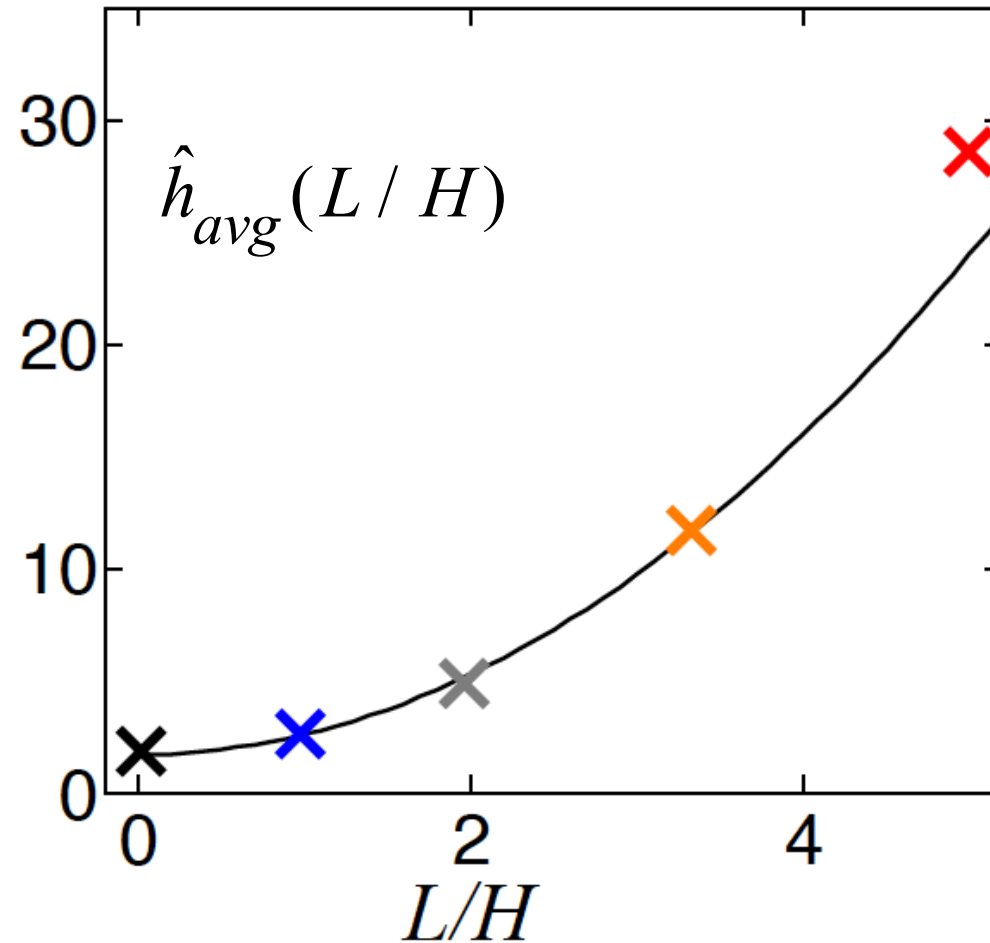

$\phi(L/H)$

For a given L/H,

$$dL/dt \propto (p_{inlet} - \sigma_o)^{7/6}$$

— $\phi(L/H) \approx 5.13[1 + 0.125(L/H) + 0.183(L/H)^2]$

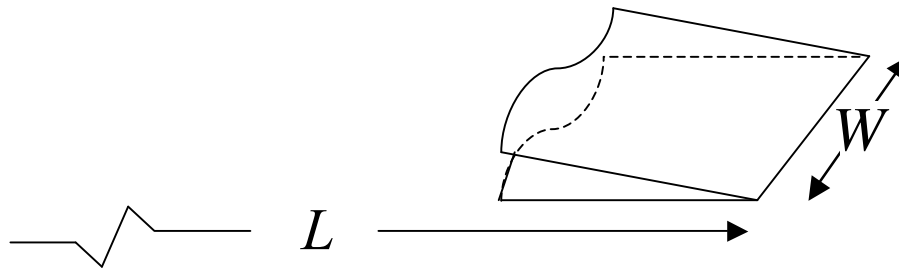**Average opening of fracture** $\equiv h_{avg} = \dfrac{(p_{inlet} - \sigma_o)L}{E'} \hat{h}_{avg}(L/H)$

$\hat{h}_{avg}(L/H)$

L/H

$\qquad \hat{h}_{avg}(L/H) \approx 1.72\left[1 + 0.517\,(L/H)^2\right]$

Volumetric inflow rate to glacier bed
(*W* ~ 2 - 3 km *is effective length perpendicular to plane*
***for use of our 2D plane - strain solution - - 3D is needed!*) :**

$$Q_{basal} = \frac{d(2LWh_{avg})}{dt} = \frac{2(p_{inlet} - \sigma_o)W}{E'} \frac{\partial[L^2 \hat{h}_{avg}(L/H)]}{\partial L} \frac{dL}{dt}$$

$$\left(\text{here}, \partial[L^2 \hat{h}_{avg}(L/H)]/\partial L \approx 3.44 L [1 + 1.035(L/H)^2] \right)$$

L W

Note : For a given L/H, $dL/dt \propto (p_{inlet} - \sigma_o)^{7/6}$,

so $Q_{basal} \propto (p_{inlet} - \sigma_o)^{13/6}$

$$p_{hydrostat} \equiv \rho_{water}gH \geq p_{inlet} \geq \rho_{ice}gH \equiv \sigma_o$$
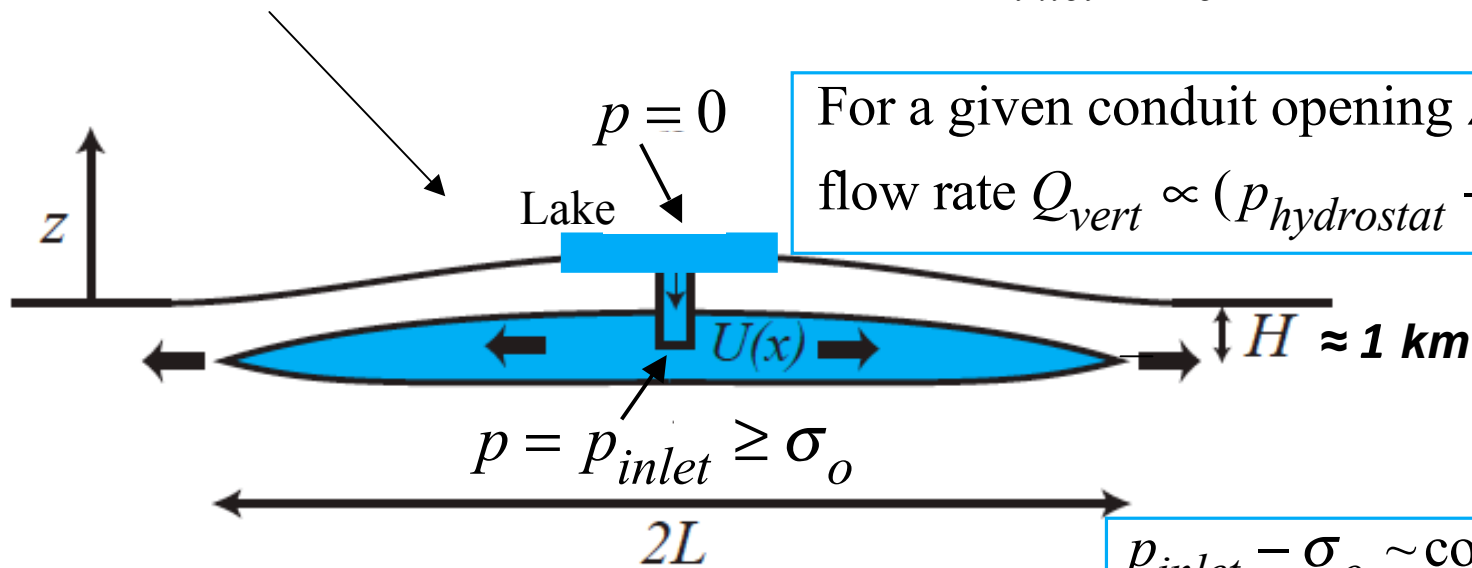
Finite L / H

Average vertical conduit opening:

$\Delta\bar{u} = \Delta\bar{u}^{el} + \Delta\bar{u}^{cr}$ (elastic + prior creep)

$\Delta\bar{u}^{el} \propto p_{inlet} - \sigma_o$

Features not to scale!

$p = 0$

Lake

z

For a given conduit opening $\Delta\bar{u}$, vertical flow rate $Q_{vert} \propto (p_{hydrostat} - p_{inlet})^{1/2}$

$U(x)$

$\updownarrow H \approx$ **1 km**

$p = p_{inlet} \geq \sigma_o$

$2L$

$p_{inlet} - \sigma_o \sim$ controls flow rate Q_{basal} into basal fracture

Schematic for turbulent hydraulic fracture

(Analysis simplified by treating ice and bed as a *homogeneous* medium, thus $h = w =$ opening.)
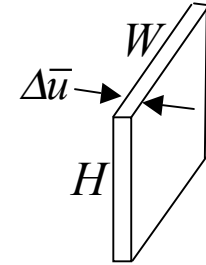
p_{inlet} ultimately determined by setting $Q_{basal} = Q_{vert}$

To evaluate the vertical crack - crevasse system as a feeder channel :

For evaluating **flow resistance**: Vertical crack-crevasse system treated as a vertical slit
of depth H, uniform width W, and uniform (but time-dependent) opening gap $\Delta \bar{u}$:

$$\Rightarrow \ Q_{vert} \approx 1.97 \left(\frac{\rho g H - p_{inlet}}{\rho g H - \sigma_o} \right)^{1/2} g^{1/2} W \, \Delta \bar{u}^{-3/2} \left(\frac{\Delta \bar{u}}{k} \right)^{1/6}$$

$\boxed{\Delta \bar{u} = \text{short-time } \textit{elastic } (\Delta \bar{u}^{el}) + \text{longer term } \textit{creep } (\Delta \bar{u}^{cr})}$

Elastic opening gap Δu^{el} calculated by 2D plane strain elasticity, $\Delta \bar{u}^{el} = \dfrac{\pi (p_{inlet} - \sigma_o)}{2E'} W$

Creep opening gap $\Delta \bar{u}^{cr}$ will be constant during the short timescale of rapid drainage.
Will depend on how long the vertical crack-crevasses system has
been hydrostatically pressurized before nucleation of basal fracture.

Power - law creep, $\dot{\gamma} / 2 = A(T)\tau^n$, where $n = 3$ (Glen's law) is typical for glacial flow.

$$\frac{d\Delta \bar{u}^{cr}}{dt} \approx \frac{\pi}{4} A(T) \left(\frac{(\rho - \rho_{ice})gH}{2n} \right)^3 W \ \text{during hydrostatic pressure loading of walls}$$

Define $\boxed{C \equiv \dfrac{\Delta \bar{u}^{cr}}{(\Delta \bar{u}^{el})_{p_{inlet} = p_{hydrostat}}}}$, **so that** $\Delta \bar{u} \approx C \dfrac{\pi (\rho g H - \sigma_o)}{2E'} W + \dfrac{\pi (p_{inlet} - \sigma_o)}{2E'} W$

$\boxed{\textbf{At} - 5°\textbf{C}, \ C = 1, \ \textbf{is reached in} \approx 13 \textbf{ hours } \textbf{ -- so we expect } C > 1 \ (> 16 \textbf{ hr of slow leakage})}$
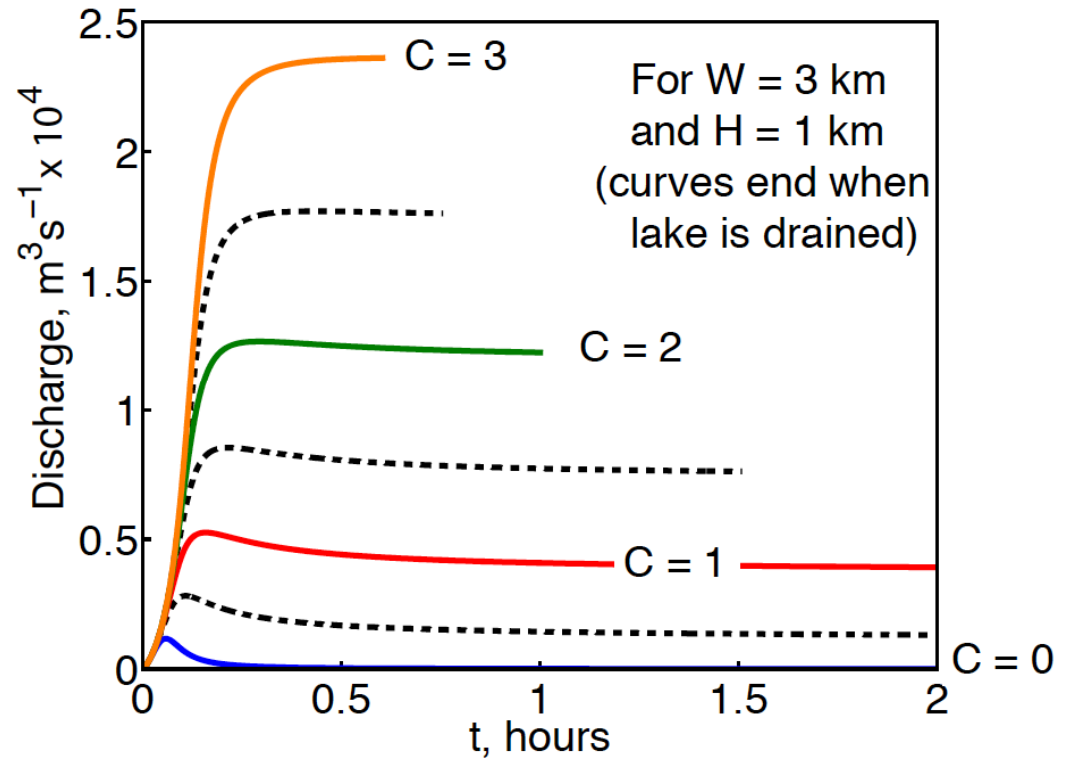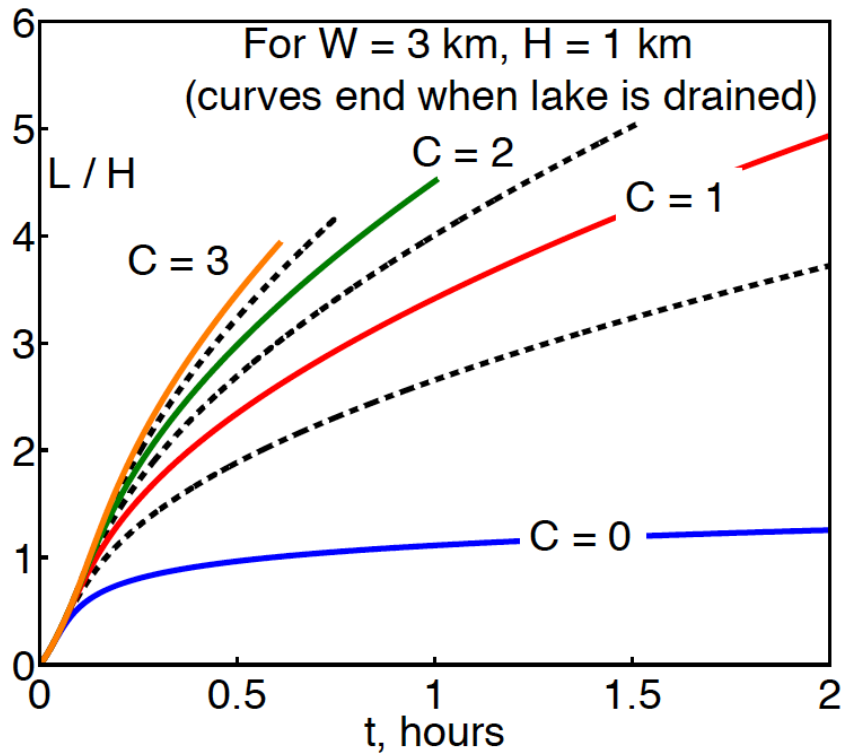
(based on $H = 1$ km, $E' = 6.8$ GPa, $n = 3$, and $A = 9 \times 10^{-25} \text{s}^{-1}\text{Pa}^{-3}$).
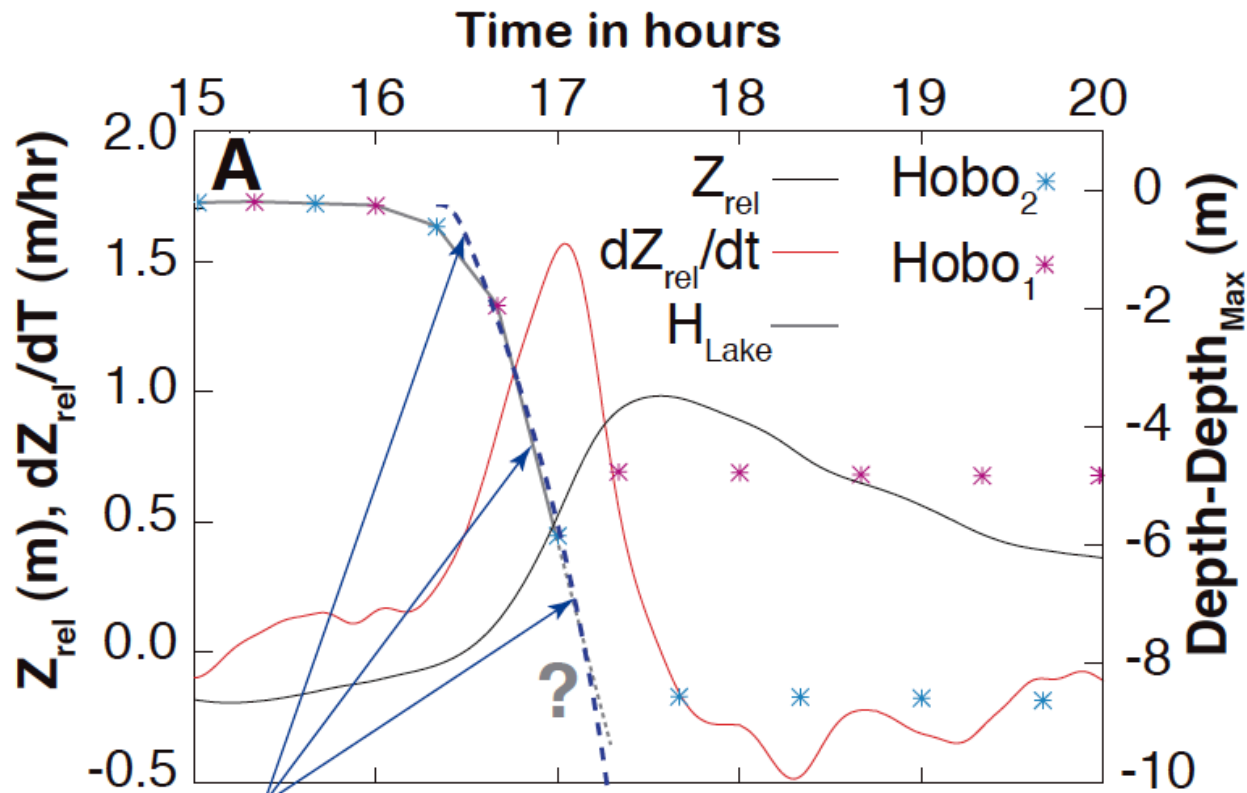
$$C = \frac{\text{prior creep opening of vertical crevasse channel}}{\text{elastic opening under hydrostatic pressurization}} = \frac{\Delta \bar{u}^{cr}}{(\Delta \bar{u}^{el})_{p=\text{hydrostatic}}}$$

$C = 1$ is achieved in ~13 hours of hydrostatic pressurization

Slow prior drainage suggests at least ~16 hours of prior high pressurization $\Rightarrow$ $C > 1$

Plots by John D. Platt:

Results of model including creep opening
of vertical crack-crevasse system, with
C = 1.75 (creep opening = 1.75 X initial
elastic opening), H = 1 km, and W = 3 km.

$C = 1.75 \Rightarrow$ Time for Drainage $= 1.21$ hr;

$$\frac{\text{Lake Volume}}{\text{Crack Area } 2WL \text{ at Drainage}} = 1.54 \text{ m}; \text{ compare}, 1.15 \text{ m GPS uplift } (Z_{rel}).$$
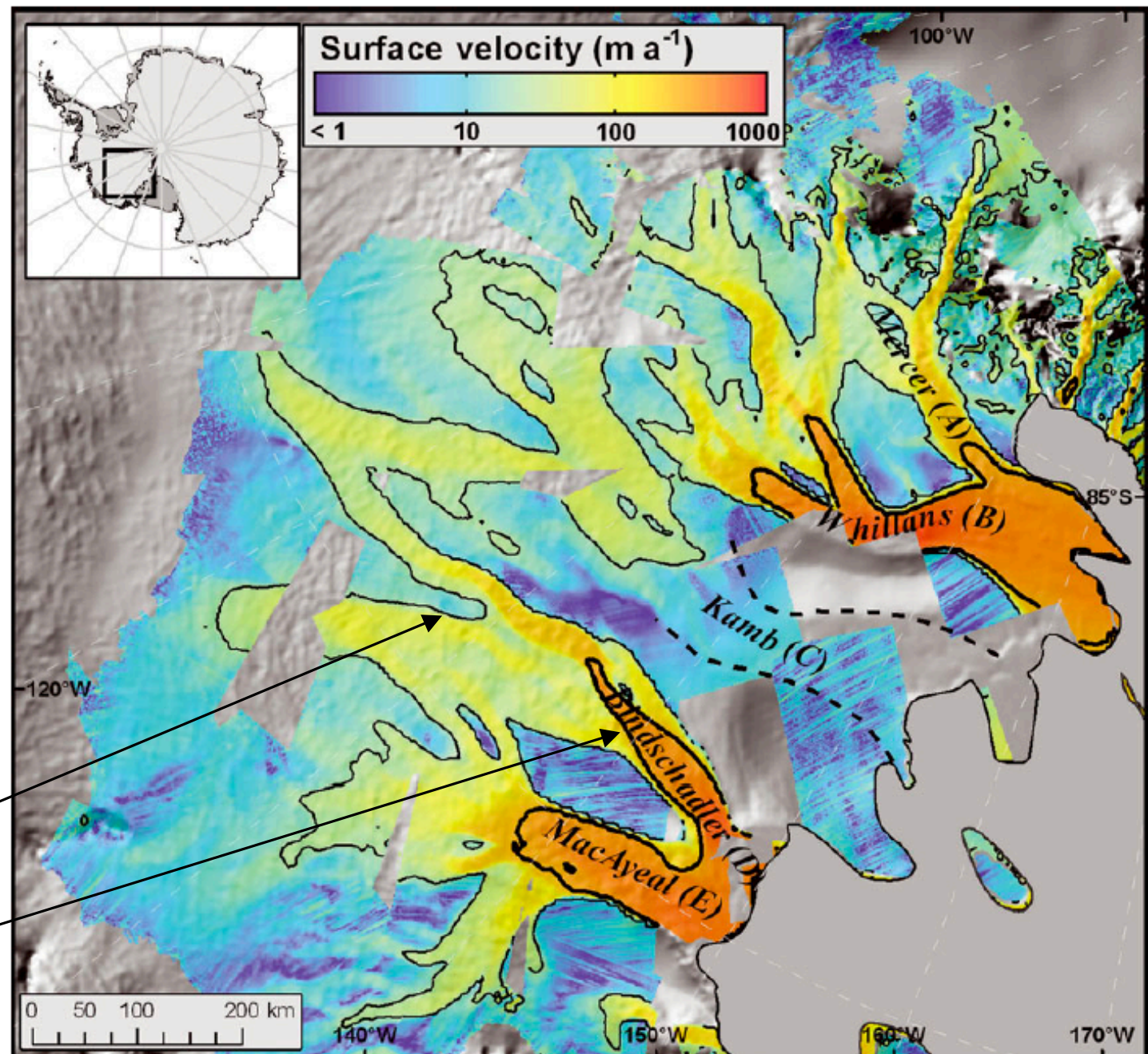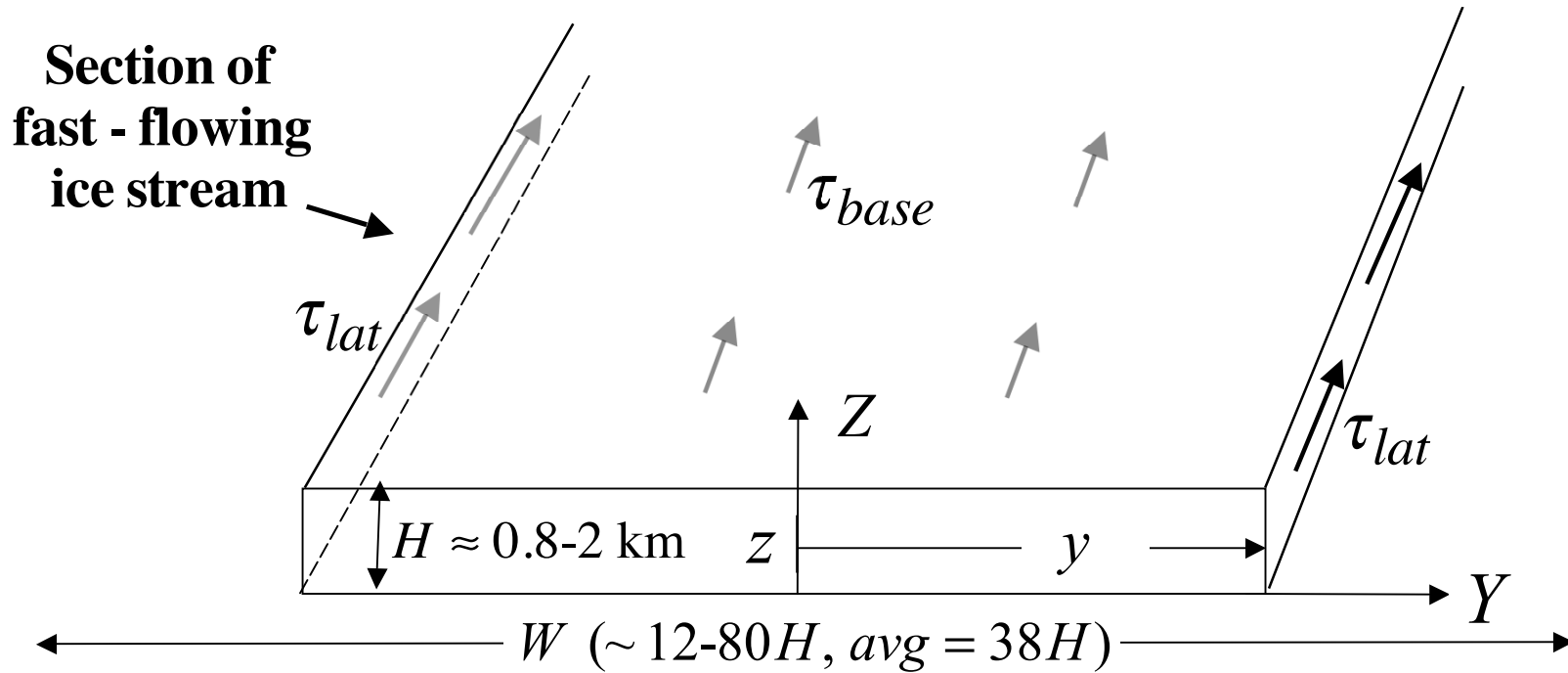
[from Le Brocq, Payne, Siegert & Alley, *J. Glac*., 2009]

• **Western Antarctica, Siple Coast,** *Ice Streams*, flowing to the Ross Sea ice shelf.

• InSAR velocity (from Joughin et al., *J. Geophys. Res.,* 2002) overlaid on a digital elevation model (Bamber et al., 2009).

• Velocity contours shown are 25 m/yr (thin line) and 250 m/yr (thick line).

Section of fast-flowing ice stream

τ_{lat}

τ_{base}

τ_{lat}

Z

$H \approx 0.8\text{-}2$ km $\quad z \quad\quad\quad y$

Y

$W \ (\sim 12\text{-}80H, \ avg = 38H)$

$\tau_{grav} = \rho g H S \ \ (S = \text{ slope})$
$= \text{downslope gravity}$
force per unit base area

Increases with stream width W $(\sim 6\text{-}40$ at current margins)

low

Equilibrium*: $\bar{\tau}_{lat} H = (\tau_{grav} - \tau_{base})_{avg} \ y$

$\bar{\tau}_{lat} \to (\tau_{grav} - \tau_{base})_{avg} \left(W / 2H \right)$ at margins

$\Rightarrow$ increasing strain rate, $\dot{\gamma}_{lat} \sim \bar{\tau}_{lat}^3 \sim (W / H)^3,$
$\Rightarrow$ increasing shear heating, $\sim (W / H)^4,$
$\Rightarrow$ onset of melting ?
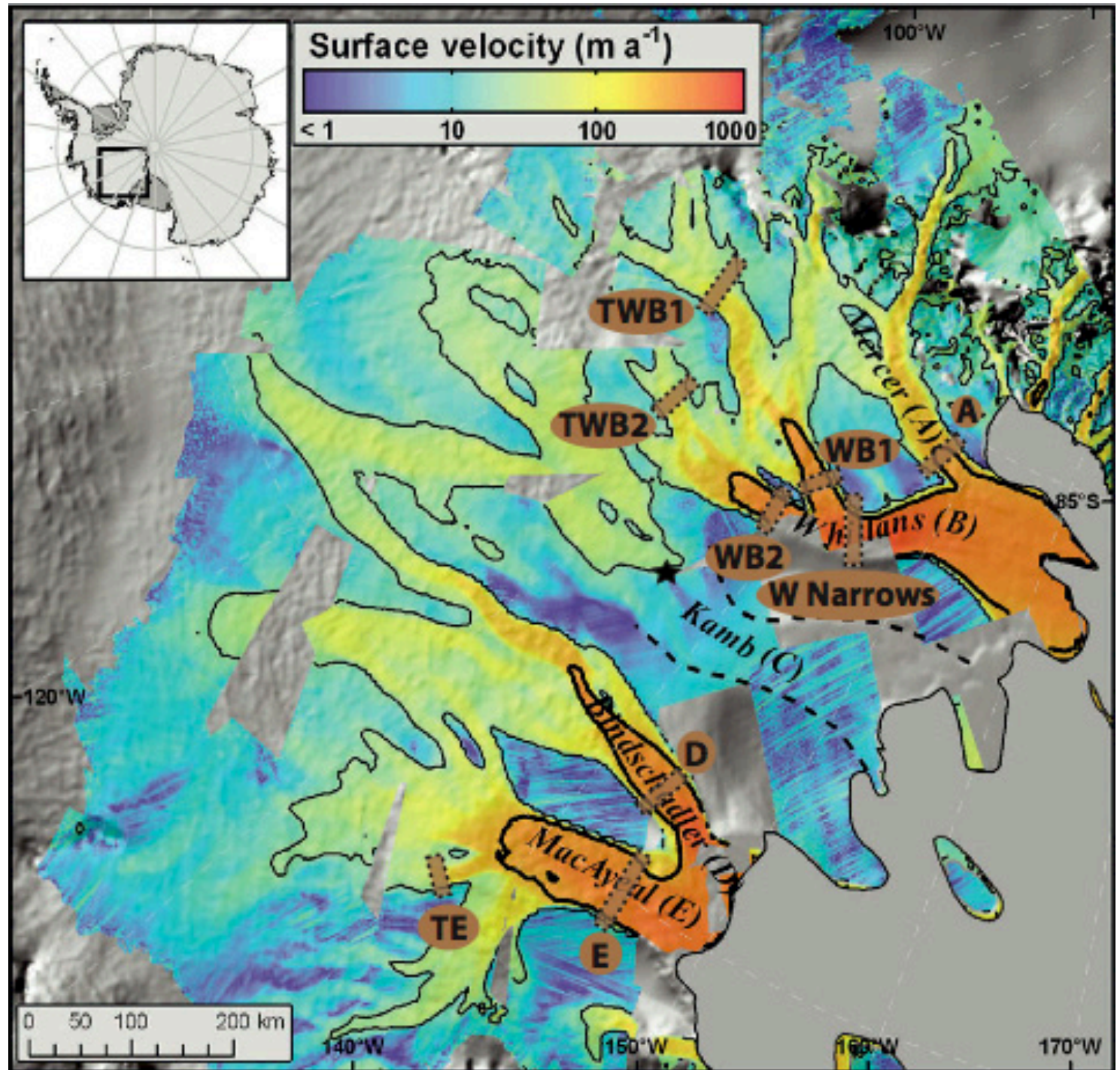
*neglecting any variation in net axial force in sheet,
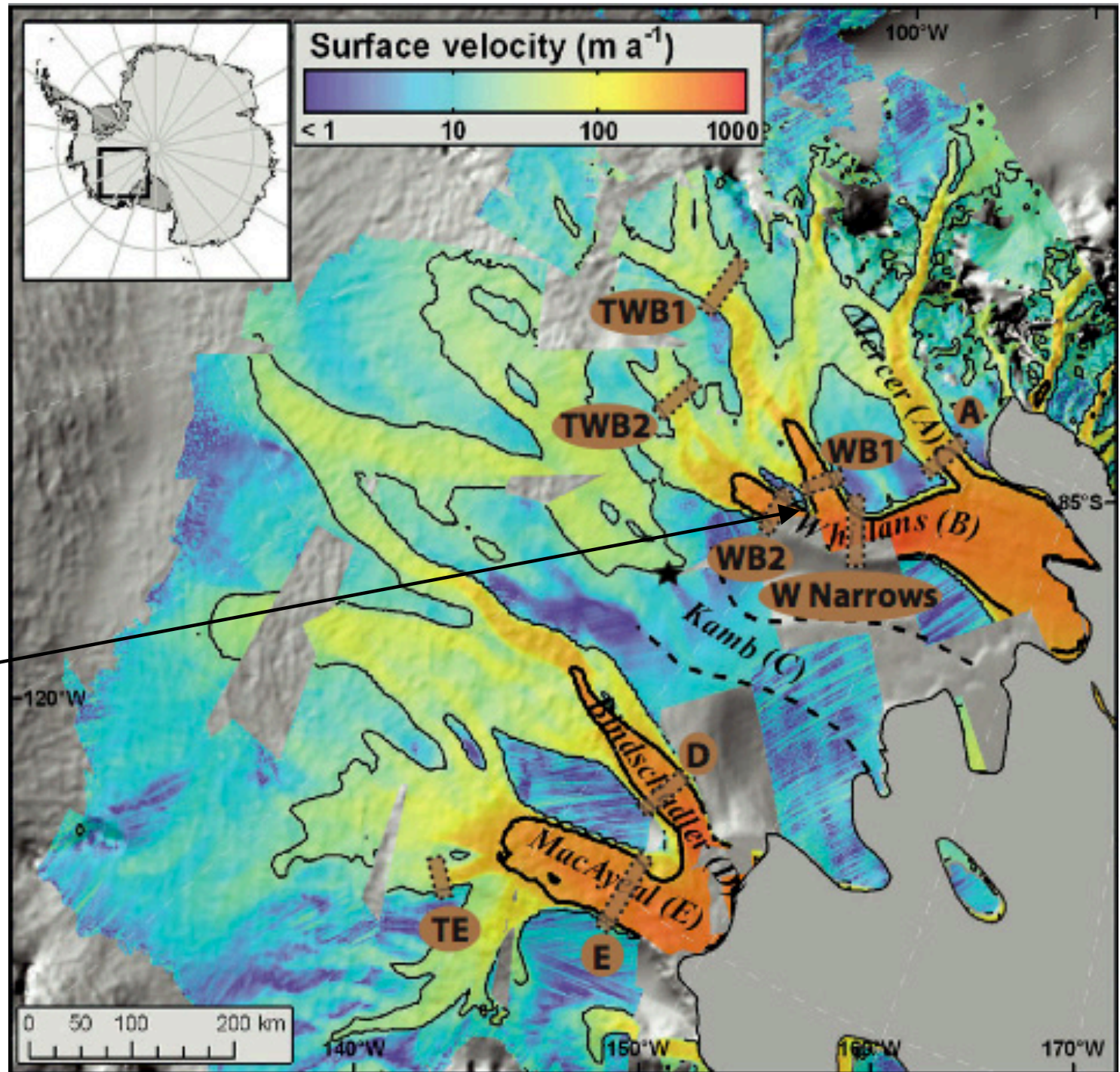roughly justified [Whillans and van derVeen, *J. Glac.*, 1993]

Nine ice stream traverses (dotted-lines) for velocity profiles measured by Joughin et al. (*J. Geophys. Res.,* 2002)
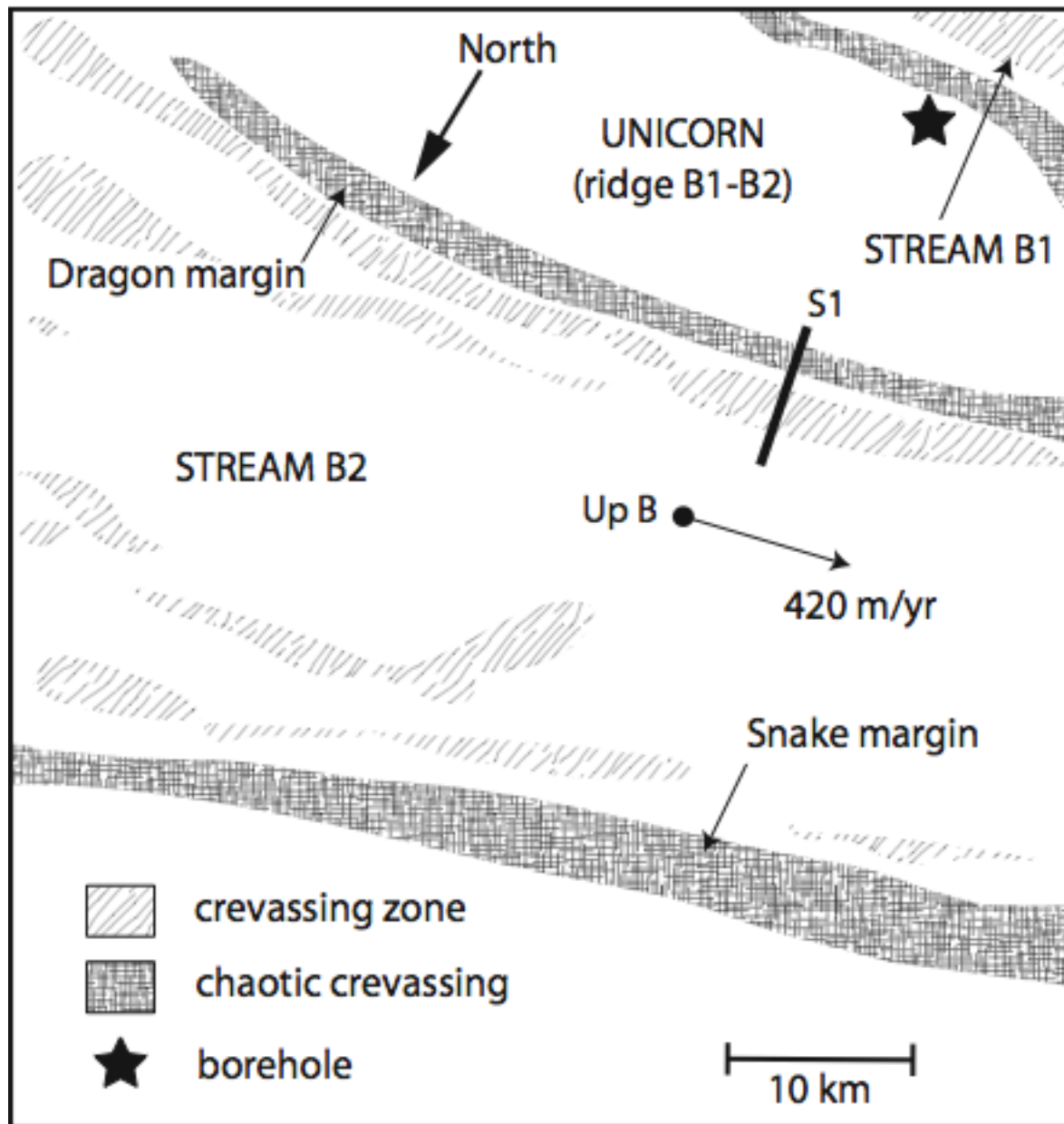
Nine ice stream traverses (dotted-lines) for velocity profiles measured by Joughin et al. (*J. Geophys. Res.*, 2002)

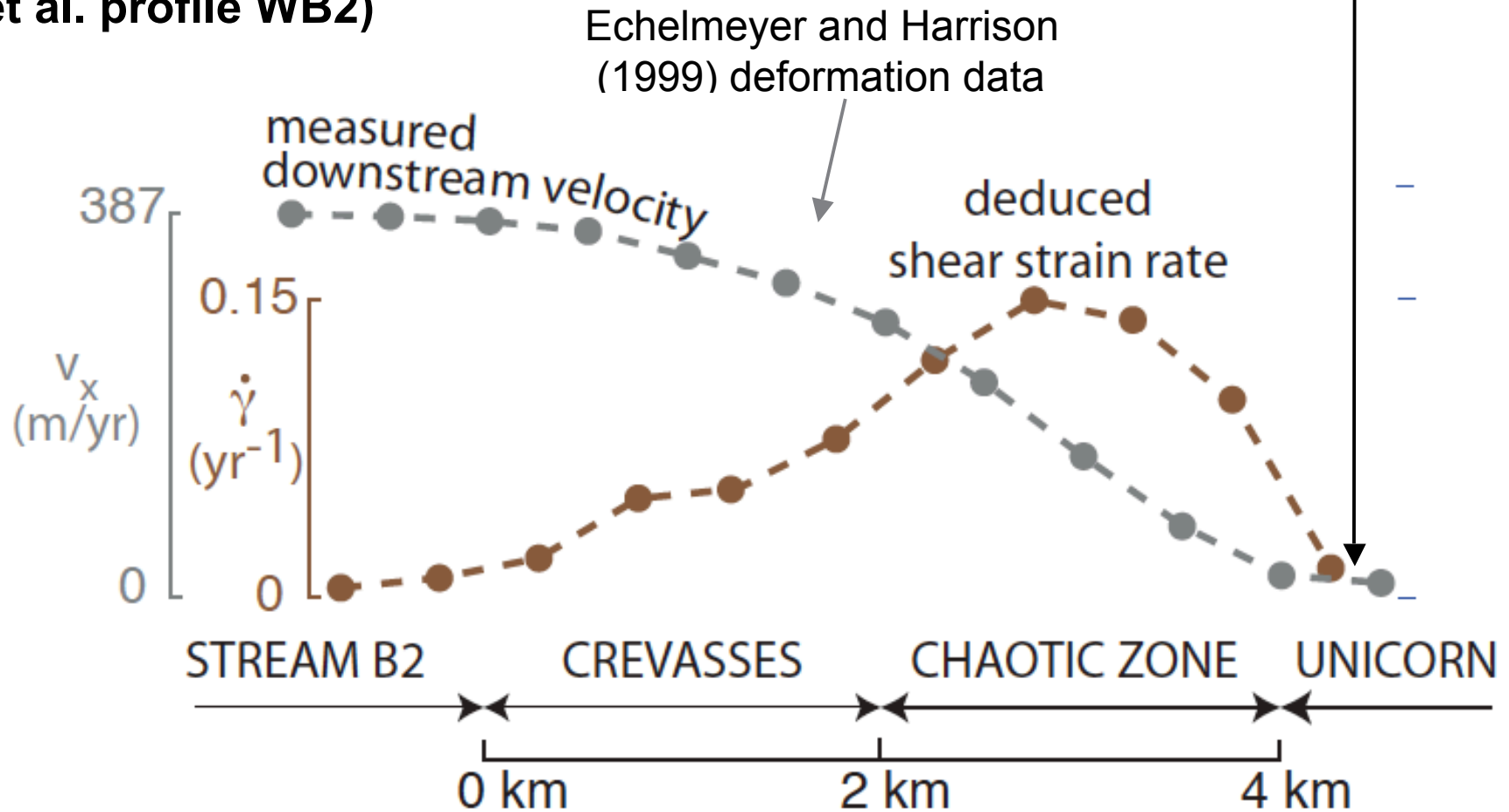Dragon Margin (near to, but not the same as, the WB2 profile)

North

UNICORN
(ridge B1-B2)

STREAM B1

Dragon margin

S1

STREAM B2

Up B ●

420 m/yr

Snake margin

crevassing zone

chaotic crevassing

★ borehole

10 km

Echelmeyer and Harrison (1999)

Dragon Margin, along Whillans upstream branch B2, profile S1 (located near Joughin et al. profile WB2)

margin of stream

Echelmeyer and Harrison (1999) deformation data

measured downstream velocity

deduced shear strain rate

387

v_x (m/yr)

0.15

$\dot{\gamma}$ (yr^{-1})

0

0

STREAM B2   CREVASSES   CHAOTIC ZONE   UNICORN

0 km   2 km   4 km

Ice Stream (Joughin et al. group)	Profile	H^a (m)	W^b (km)	T_{melt}^c (° C)	$\dot{\gamma}_{lat}^d$ (10^{-2}.yr^{-1})
Dragon Margin		985	34		15
Mercer	A	1242	39	−0.8	4.2
Whillans	WB1	1205	35	−0.8	7.0
	WB2	985	34	−0.6	9.5
	W Narrows	846	48	−0.6	13.5
	TWB1	2188	25	−1.5	3.8
	TWB2	1538	25	−0.9	4.0
Bindschadler	D	888	55	−0.6	5.8
MacAyeal	E	916	78	−0.6	8.1
	TE	1177	19	−0.8	5.5

Tributaries

From Joughin et al. [2002]:
a. Ice sheet thickness H; b. Ice stream width W; d. Maximum shear strain rate at margin, $\dot{\gamma}_{lat}$.

c. Melting temperature at bed. When allowing for a temperate zone in modeling, we set this temperature to be the uniform melting temperature over the zone.

Temperature distribution implied at West Antarctic Ice Stream margins.

1-D Model: Neglecting horizontal (but not vertical) advective ice motions, and horizontal T gradients at the margins, and considering only $\dot{\gamma}_{lat}$, the temperature distribution through the column of ice at *steady state* $(\partial T(z,t)/\partial t = 0;\ T = T(z))$ satisfies:

$$\frac{d}{dz}\left(K(T)\frac{dT}{dz} \right) + \tau_{lat}(\dot{\gamma}_{lat},T)\dot{\gamma}_{lat} = \rho C_i(T)w\frac{dT}{dz}$$

- We take $\dot{\gamma}_{lat}$ to be ***uniform*** in depth, with ***Glen's law*** giving

$$\tau_{lat}(\dot{\gamma}_{lat},T) = \left(1/\ A(T)\right)^{1/3}(\dot{\gamma}_{lat}/2)^{1/3},\ \text{ and write } \textit{vertical velocity } w$$

as $w = -az/H$ (Zotikov form, where $a \approx$ surface accumulation rate).

- Thus, ***with solutions constrained by*** $T \le T_{melt}$, $T(z)$ is given by

$$\frac{d}{dz}\left(K(T)\frac{dT}{dz} \right) + 2\left(1/\ A(T)\right)^{1/3}(\dot{\gamma}_{lat}/2)^{4/3} = -\rho C_i(T)\left(\frac{az}{H} \right)\frac{dT}{dz}$$

Thermo-mechanical properties of ice

Data fits as suggested by Cuffy & Paterson [2010]

Glen's flow law for ice (dislocation creep):

$$\dot{\gamma}/2 = A(T)\tau^3,$$

$$\tau = B(T)(\dot{\gamma}/2)^{1/3}$$

$$\left(B(T) = [1/A(T)]^{1/3}\right)$$

With diffusion creep too,

$$\dot{\gamma}/2 = C(T,d_g)\tau + A(T)\tau^3$$

Ice is still strong at T_{melt}:

For a given $\dot{\gamma}$:
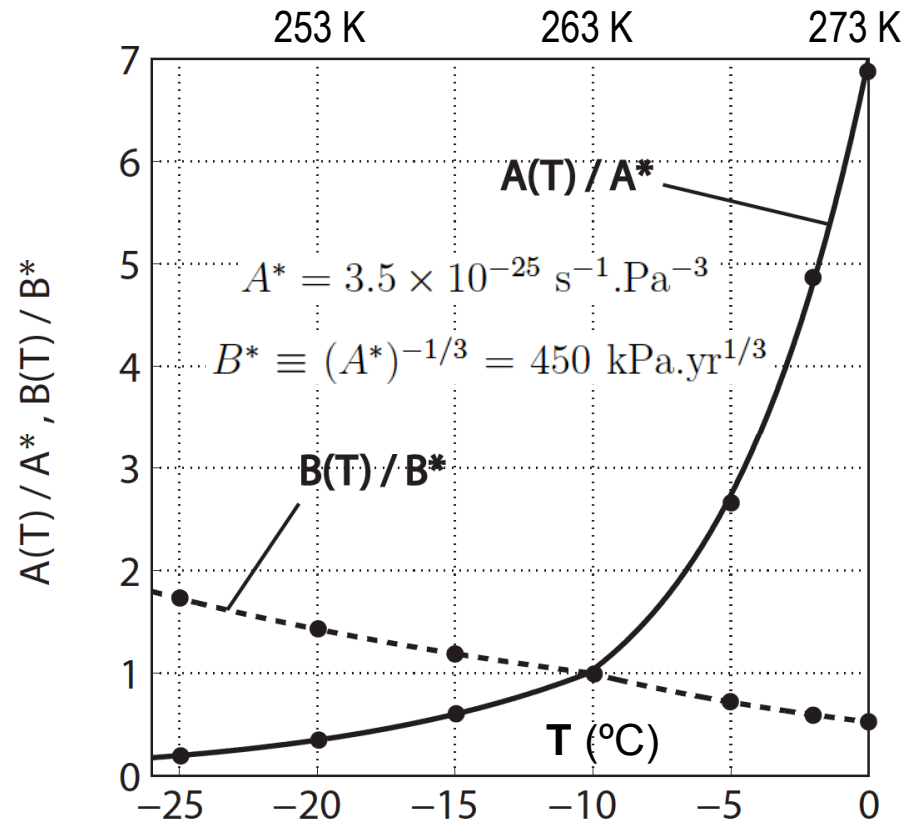
$$\tau_{T=0°C} \approx 0.5 \times \tau_{T=-13°C}$$

Graph: vertical axis $A(T)/A^*$, $B(T)/B^*$ (0 to 7); horizontal axis T (°C) from -25 to 0; top axis 253 K, 263 K, 273 K.

$A^* = 3.5 \times 10^{-25}$ s^{-1}.Pa^{-3}

$B^* \equiv (A^*)^{-1/3} = 450$ kPa.yr$^{1/3}$

Curves labelled $A(T)/A^*$ and $B(T)/B^*$.

Thermal conductivity:

$$K(T) = 9.828 \frac{J}{m\,s\,K} \exp\left(-5.7 \times 10^{-3}\frac{T}{K}\right)$$

Specific heat:

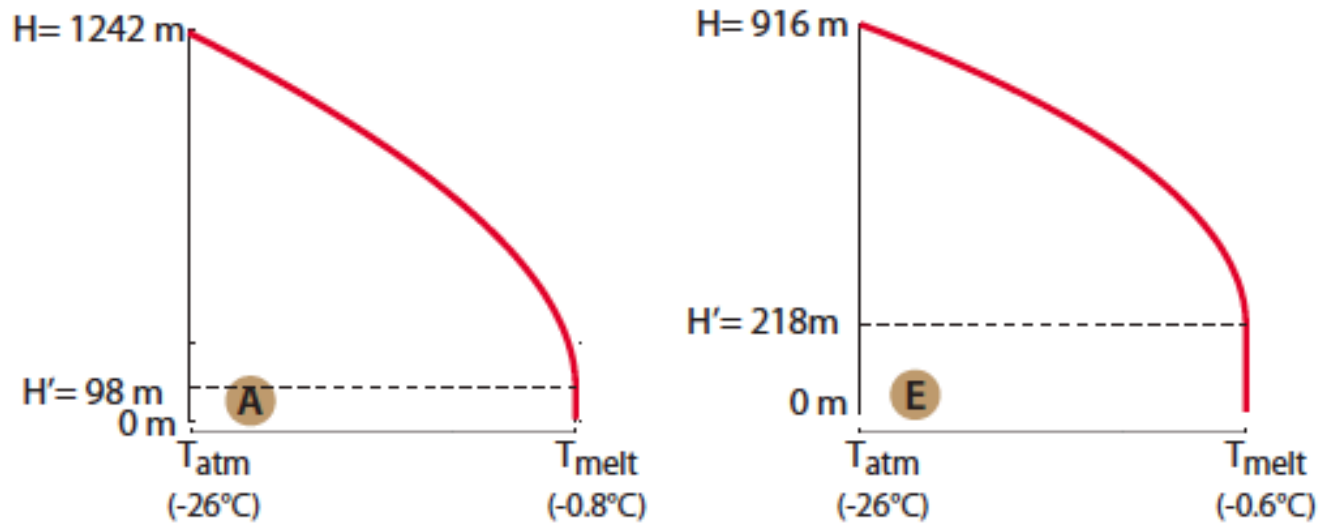$$C_i(T) = \left(152.5 + 7.122\frac{T}{K}\right)\frac{J}{kg\,K}$$

$$\frac{d}{dz}\left(K(T)\frac{dT}{dz}\right)+\left(\frac{1}{2A(T)}\right)^{1/3}(\dot{\gamma}_{lat})^{4/3}+\rho C_i(T)(az/H)\frac{dT}{dz}=0 \;\; \& \;\; T\leq T_{melt} \;\;\Rightarrow$$

H= 846 m

H'= 367m

W Narrows

0 m

T_{atm}
(-26°C)

T_{melt}
(-0.6°C)

H= 1205 m

H'= 451m

WB1

0 m

T_{atm}
(-26°C)

T_{melt}
(-0.8°C)

H= 985 m

H'= 381m

WB2

0 m

T_{atm}
(-26°C)

T_{melt}
(-0.6°C)

(For all cases,
a = 0.1 m/yr)

H= 1242 m

H'= 98 m
0 m

A

T_{atm}
(-26°C)

T_{melt}
(-0.8°C)

H= 916 m

H'= 218m

0 m

E

T_{atm}
(-26°C)

T_{melt}
(-0.6°C)
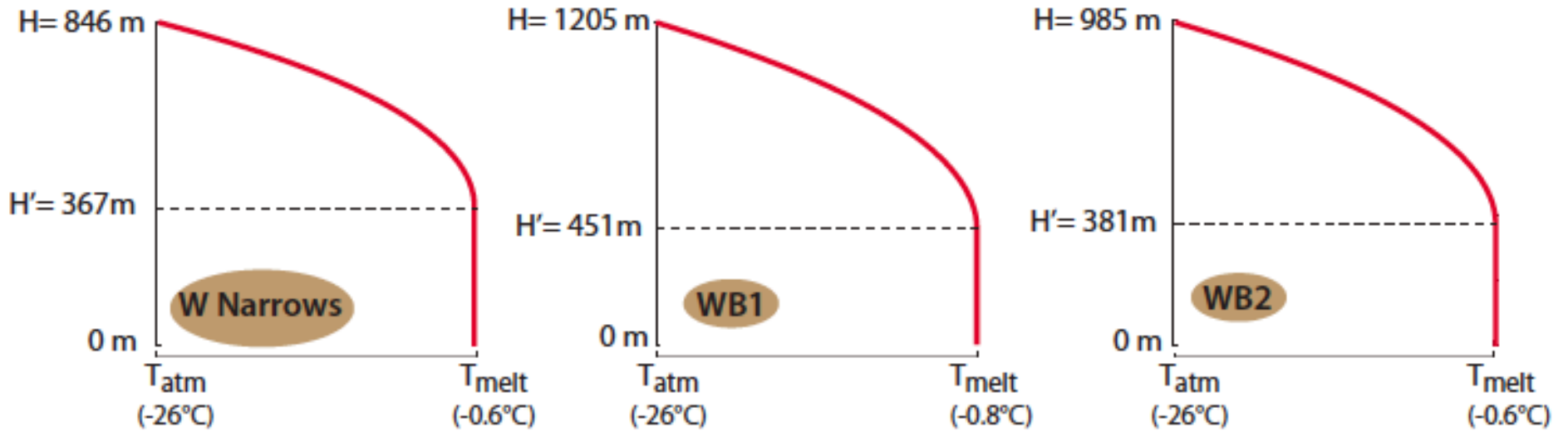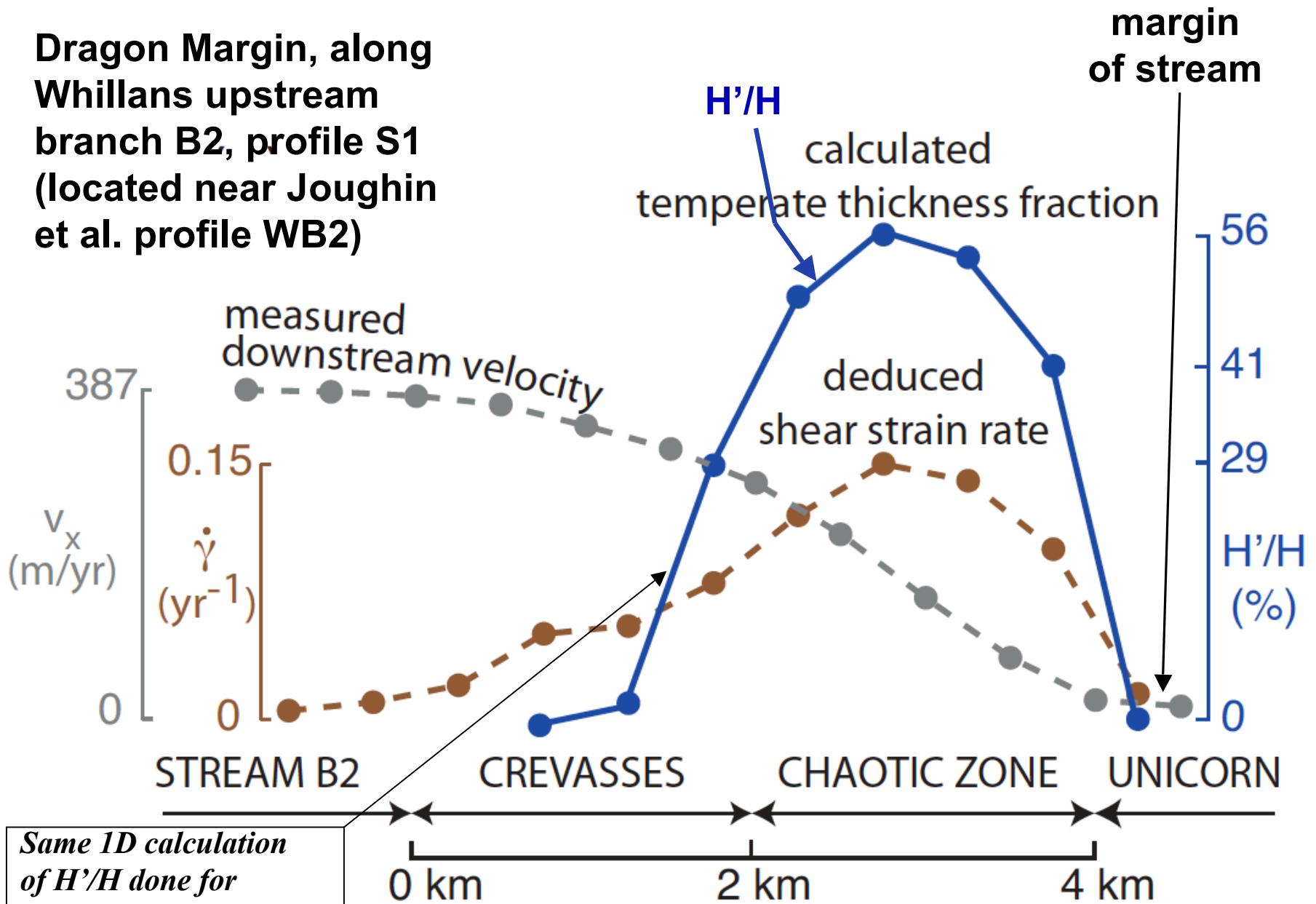
Results of calculations of $T = T(z)$ at margins for the **major ice streams**:

% of margin that is temperate, & $\bar{\tau}_{lat} = \dfrac{1}{H} \displaystyle\int_0^H \tau_{lat}(z)dz$ with $\tau_{lat}(z) = \left(\dfrac{\dot{\gamma}_{lat}}{2A(T(z))} \right)^{1/3}$

Ice Stream	Profile	τ_{grav}[a](kPa)	H'/H [b](%)	$\bar{\tau}_{lat}$ (kPa)	τ_{base} (kPa)	τ_{base}/τ_{grav} (%)
Dragon Margin		11.0	56	132.0	3.2	29
Mercer	A	14.9	8	119.0	7.7	52
Whillans	WB1	12.5	37	115.6	4.5	36
	WB2	10.8	39	127.0	3.4	32
	W Narrows	7.6	43	138.5	2.7	36
Bindschadler	D	10.0	0[c]	131.7	5.7	58
MacAyeal	E	15.3	24	128.9	12.3	80

[a] Inferred by *Joughin et al.* [2002], $\tau_{grav} = \rho_{ice}gHS$ where S is the downstream slope.

[b] Predicted temperate height fraction of the ice sheet at margin.

[c] The ratio increases to $\sim 53\,\%$ when evaluated 30 km downstream (see text).

Results of calculations of $T = T(z)$ at margins for the **upstream tributaries**:

% of margin that is temperate, & $\bar{\tau}_{lat} = \dfrac{1}{H}\displaystyle\int_0^H \tau_{lat}(z)dz$ with $\tau_{lat}(z) = \left(\dfrac{\dot{\gamma}_{lat}}{2A(T(z))}\right)^{1/3}$

Ice Stream	Profile	τ_{grav} [a] (kPa)	H'/H [b] (%)	$\bar{\tau}_{lat}$ (kPa)	τ_{base} (kPa)	τ_{base}/τ_{grav} (%)
Dragon Margin		11.0	56	132.0	3.2	29
Whillans	TWB1	47.5	50	90.8	31.6	67
	TWB2	40.9	26	101.9	28.4	69
MacAyeal	TE	44.9	23	113.6	30.8	69

Dragon Margin, along Whillans upstream branch B2, profile S1 (located near Joughin et al. profile WB2)

margin of stream

H'/H

calculated temperate thickness fraction

measured downstream velocity

deduced shear strain rate

387

0.15

v_x (m/yr)

$\dot{\gamma}$ (yr^{-1})

H'/H (%)

56

41

29

0

STREAM B2   CREVASSES   CHAOTIC ZONE   UNICORN

Same 1D calculation of H'/H done for various locations, with $\dot{\gamma}_{lat}$ taken as the local $\dot{\gamma}$

0 km   2 km   4 km

Echelmeyer and Harrison (1999) deformation data

Mass rate of melt production per unit volume in temperate zone:

$$\dot{m} = \frac{\tau_{lat}\dot{\gamma}_{lat}}{L} \quad , \quad \tau_{lat} = \left(\frac{\dot{\gamma}_{lat}}{2A_{melt}}\right)^{1/3} F(n) \quad (\text{where } F(n) \le 1) \quad , \quad \dot{m} = \frac{\dot{\gamma}_{lat}^{4/3} F(n)}{(2A_{melt})^{1/3} L}$$

Latent heat per unit mass   Porosity   Strength reduction due to porosity

Darcy seepage flux of melt water:

$$\vec{\nabla} \bullet (\rho_w \vec{q}) = \dot{m} \quad \Rightarrow \quad \frac{dq_z}{dz} = \frac{\dot{m}}{\rho_w} = \frac{\dot{\gamma}_{lat}^{4/3} F(n)}{(2A_{melt})^{1/3} L\rho_w} \quad [\text{assumes } \vec{q} = (0,0,q_z)]$$

$$\Rightarrow \quad (q_z)_{z=0} = -\left[\frac{1}{H'}\int_0^{H'} F(n)\,dz\right] \frac{\dot{\gamma}_{lat}^{4/3} H'}{(2A_{melt})^{1/3} L\rho_w}$$

(we assume this is ~ 1 -- see next page)

$$\text{Also, for } F(n) \approx 1, \quad q_z = -\frac{\dot{\gamma}_{lat}^{4/3} (H' - z)}{(2A_{melt})^{1/3} L\rho_w}$$

Water permeation through the partially melted ice:

J. F. Nye and F. C. Frank,
[*J. Glac.*, 1973], building on
Frank's [*Nat.*, 1968]
analysis of melt convection
in Earth's mantle:

Permeability

$$k = \alpha n^2 d_g^2$$

$n =$ porosity,

$d_g =$ grain size (1-10 mm),

$$\alpha \approx 1/2000 \text{ to } 1/1500$$
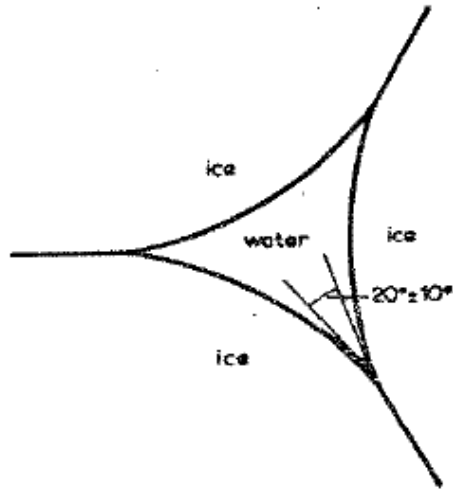
Fig. 2. Cross-section of a vein of liquid situated at a grain edge, where three grain boundaries meet. The figure is drawn for a dihedral angle φ equal to 20°, as measured for ice—water by Ketcham and Hobbs (1969).
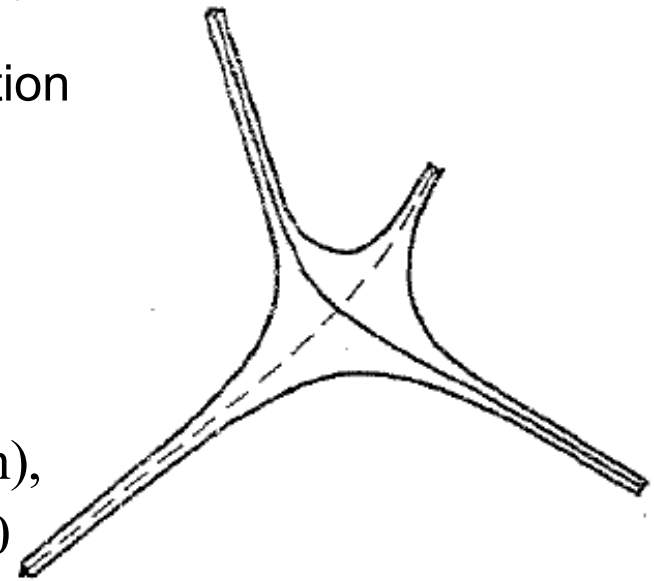
Fig. 3. A junction between four water veins in polycrystal-line ice. The figure is a tetrahedron with non-spherical faces and with open corners.

$$q_z = - \frac{\dot{\gamma}_{lat}^{4/3}(H'-z)}{(2A_{melt})^{1/3} L\rho_w} = -\frac{k}{\mu_w}\left(\frac{dp}{dz} + \rho_w g\right)$$

$$= -\frac{k}{\mu_w}(\rho_w - \rho_{ice})g \text{ if } p = \text{ ice overburden pressure} = \rho_{ice} g(H-z)$$

$$\Rightarrow \text{ porosity } n \leq 5\times 10^{-4} \text{ mm}/d_g \leq 5\times 10^{-4}$$

Dragon Margin, along Whillans upstream branch B2, profile S1

(a.)

calculated temperate thickness fraction

measured downstream velocity

deduced shear strain rate

v_x (m/yr) $\dot{\gamma}$ (yr^{-1}) H'/H (%)

387

0.15

0

0

56

41

29

0

STREAM B2   CREVASSES   CHAOTIC ZONE   UNICORN

0 km   2 km   4 km

(b.)

26

Water Darcy flux at bed

q_w (mm/yr)

41 m^2.yr^{-1}

0

STREAM B2   CREVASSES   CHAOTIC ZONE   UNICORN

0 km   2 km   4 km
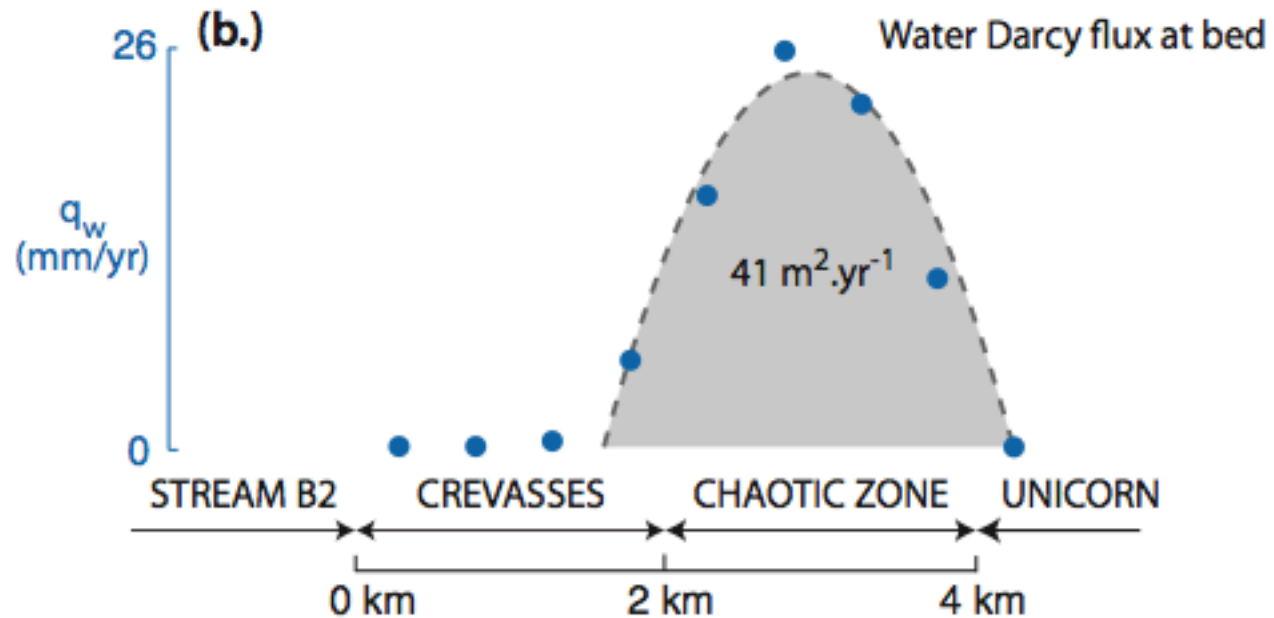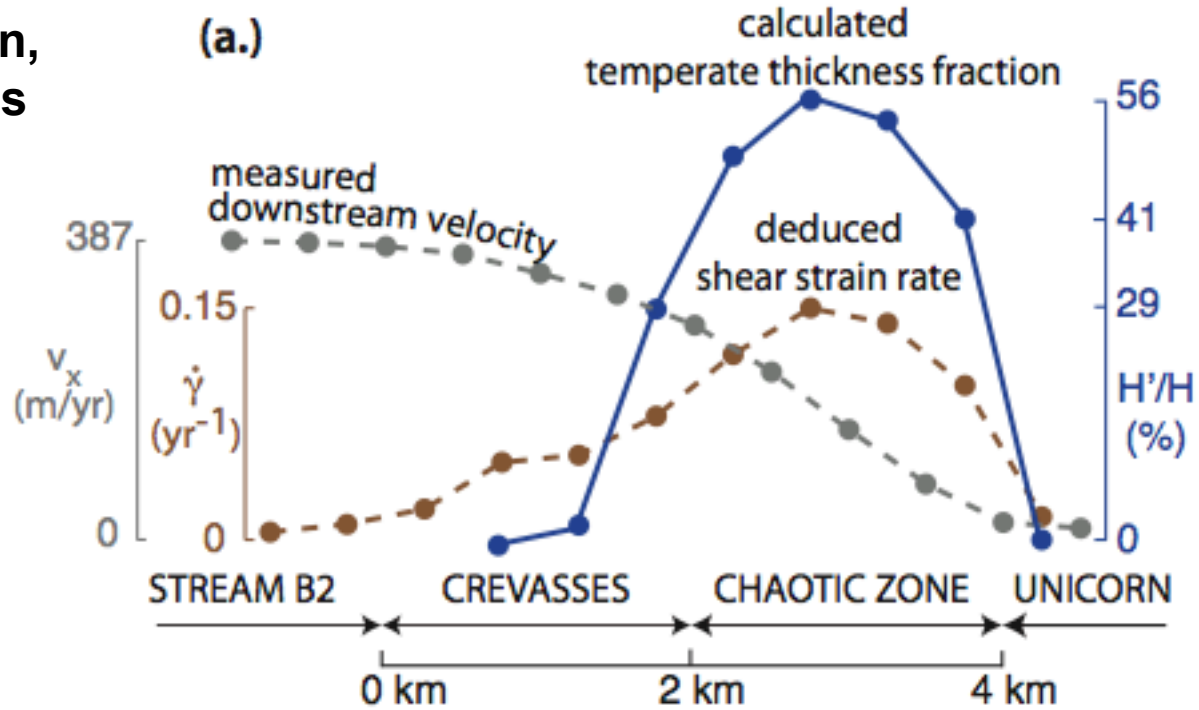
Perol and Rice, (AGU Abstr, 2011)

2D, antiplane strain analysis : $\quad$ velocity $= u(y,z)\vec{e}_x$, $\quad \dot{\gamma} = \sqrt{\overrightarrow{\vec{\nabla} u \cdot \vec{\nabla} u}}$

Coupled non - linear Poisson equation system
(for velocity u and temperature T) **:**

$$\frac{\partial}{\partial y}\left(\frac{\tau(\dot{\gamma},T)}{\dot{\gamma}}\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\tau(\dot{\gamma},T)}{\dot{\gamma}}\frac{\partial u}{\partial z}\right) = -\rho g S \qquad \left(\tau(\dot{\gamma},T) = \left(\frac{\dot{\gamma}}{2A(T)}\right)^{1/3}\right)$$

$$\frac{\partial}{\partial y}\left(K(T)\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K(T)\frac{\partial T}{\partial z}\right)$$

$$= -[1 - \hat{H}(T - T_{melt})]\tau(\dot{\gamma},T)\dot{\gamma} + \rho C(T)\left(v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right)$$

(here, v & w are regarded as given, e.g., Zotikov's $w = -az / H$, $v =$ const., v_{o})

Computational Approach (Suckale, Platt, Perol, Rice, to JGR, 2013) :
- **Multigrid methodology** for iterative solution of coupled
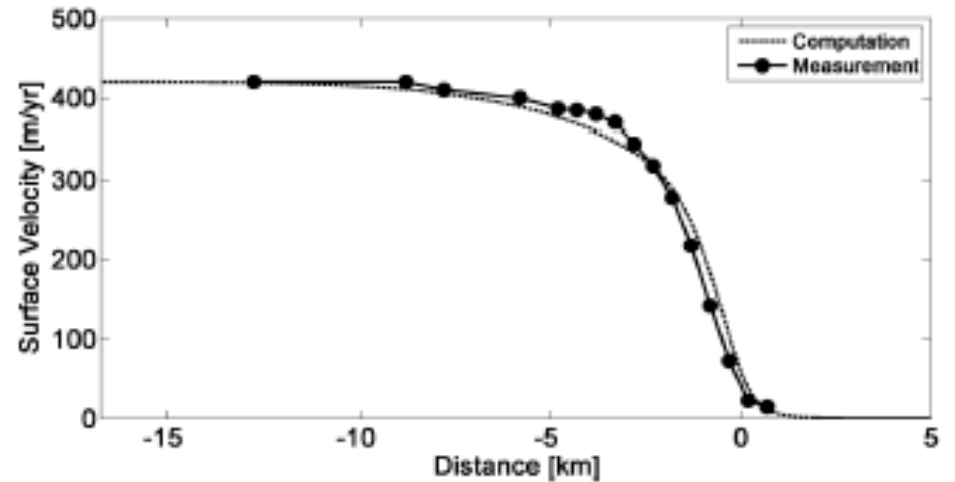 nonlinear Poisson systems, embedding constraint $T \le T_{melt}$.

Figure 8. Temperature fields and surface velocities for Dragon margin when including only vertical advection (A1 and B1) with $a = 0.1\,\mathrm{m/yr}$ and only horizontal advection (A2 and B2) with $v = -7.3\,\mathrm{km/yr}$, respectively. The best fitting basal stresses are $\tau_{base} = 5.31\,\mathrm{kPa}$ (A1 and B1) and $\tau_{base} = 0.94\,\mathrm{kPa}$ (A2 and B2), respectively. Both computations neglect surface crevassing.

A1. Temperature field (vertical advection only)
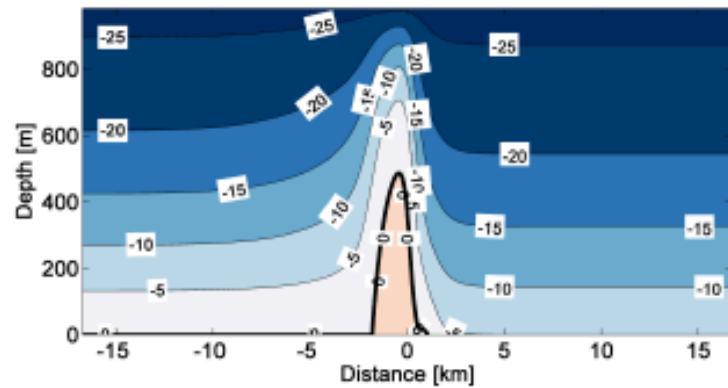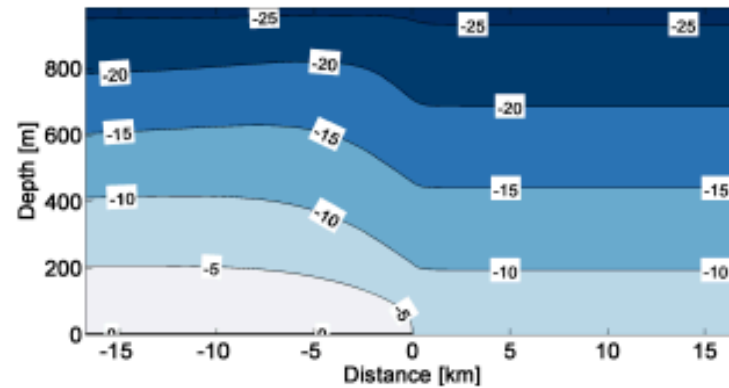
B1. Surface velocities (vertical advection only)
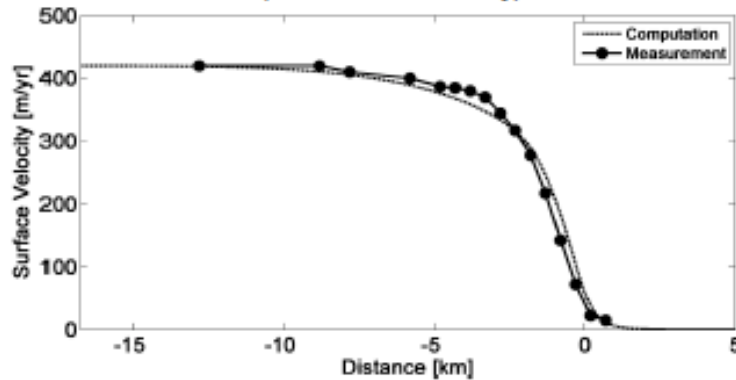
Figure 8. Temperature fields and surface velocities for Dragon margin when including only vertical advection (A1 and B1) with $a = 0.1\,\mathrm{m/yr}$ and only horizontal advection (A2 and B2)
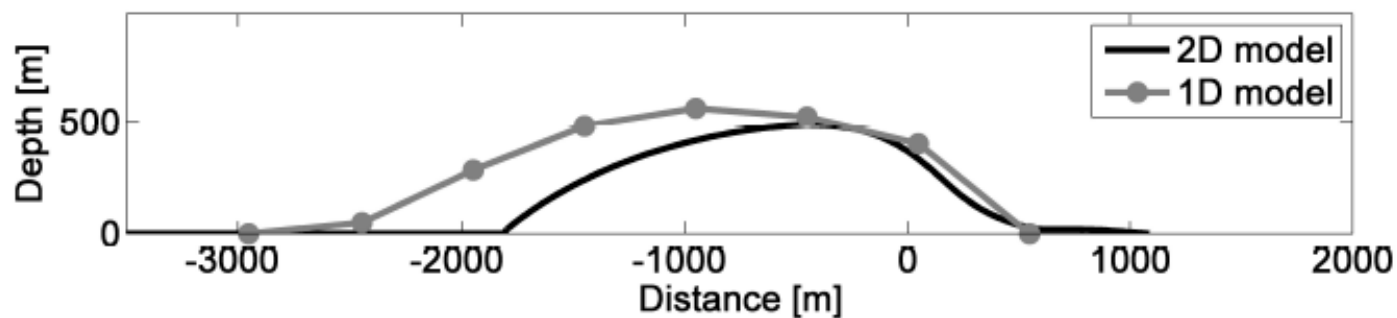
Figure 9. Comparison of the temperate zone from Fig. 8A1 and B1, replotted on a 1:1 scale, with the simplified 1D model by *Perol and Rice* [2011] using measured surface velocities
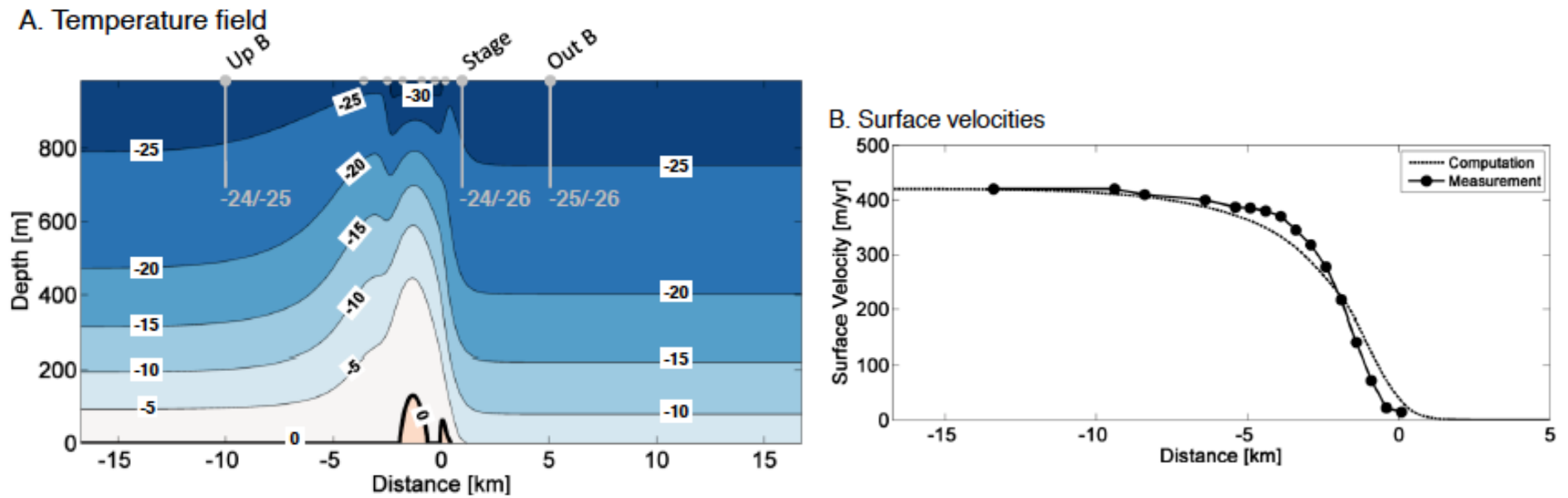
Figure 10. Temperature field (A) and surface velocities (B) for Dragon when attempting to match the observed borehole temperatures [*Harrison et al.*, 1998], see Figs. 1 and 2 for borehole locations, and surface velocities [*Echelmeyer and Harrison*, 1999] simultaneously. The computation is based on the model parameters $\tau_{base} = 1.22\,\text{kPa}$, accumulation $a = 0.23\,\text{m/yr}$, geothermal heat flux $G = 85\,\text{mW/m}^2$, and horizontal advection at $v = -0.35\,\text{m/yr}$. The approximate locations of the nine boreholes considered in *Harrison et al.* [1998] are indicated as grey dots. We highlight the boreholes located far from margin as grey lines in accordance to their depth. The left numbers represent the computed value and the right number the measured value at maximum depth.
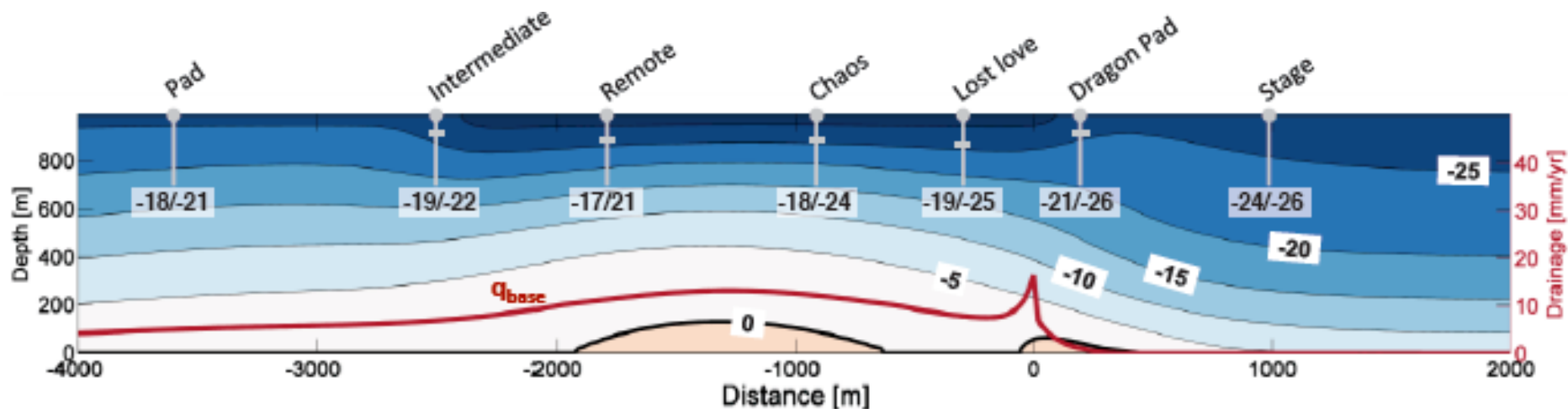
Figure 11. Extent of a potential temperate zone at Dragon margin plotted on a 1:1 scale and meltwater flux at the base of the ice, q_{base}, in mm/yr (grey line) for the computation also shown in Fig. 10. The total meltwater produced in the temperate zone is $25\,\mathrm{m^2/yr}$. The approximate locations of the boreholes from *Harrison et al.* [1998] are highlighted in grey with the left number representing the computed temperature and the right value the measured temperature at a depth of approximately $700\,\mathrm{m}$. Small horizontal dashes along the boreholes in the vicinity of the shear margin indicate the approximate position of the $-26°\mathrm{C}$ contour.

A. Error in reproducing observational data
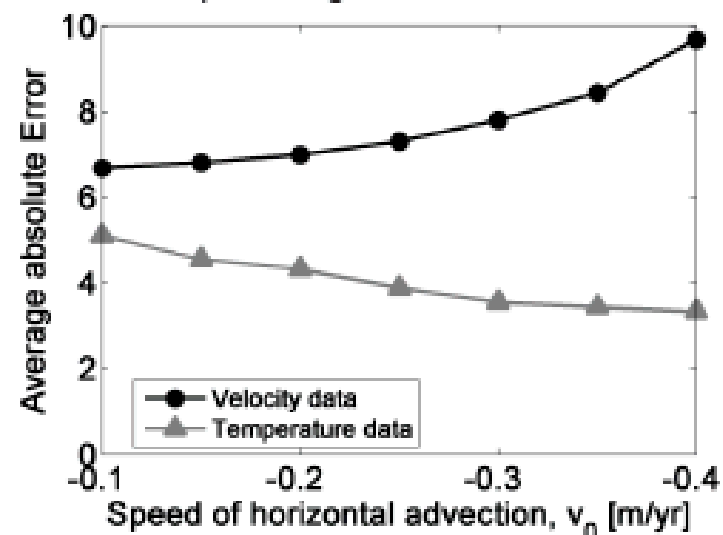
B. Properties of the temperate zones
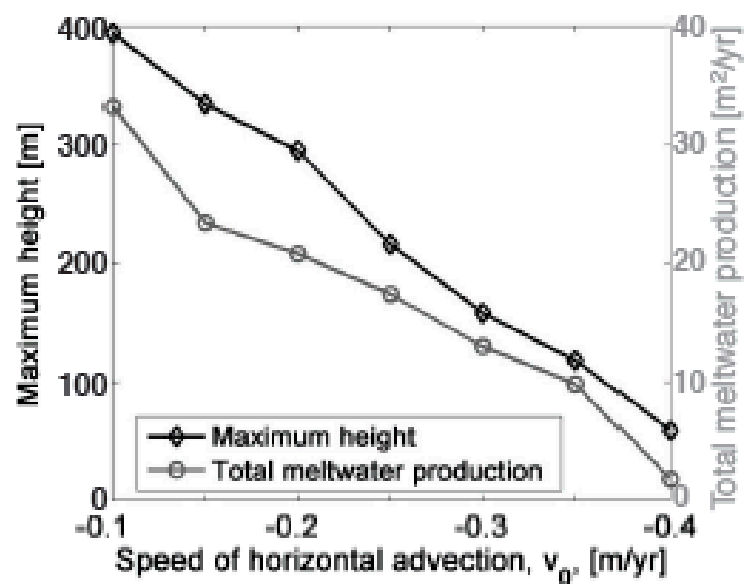
C. Temperature and drainage for specific speeds of horizontal advection

Rothlisberger-Shreve channel analysis
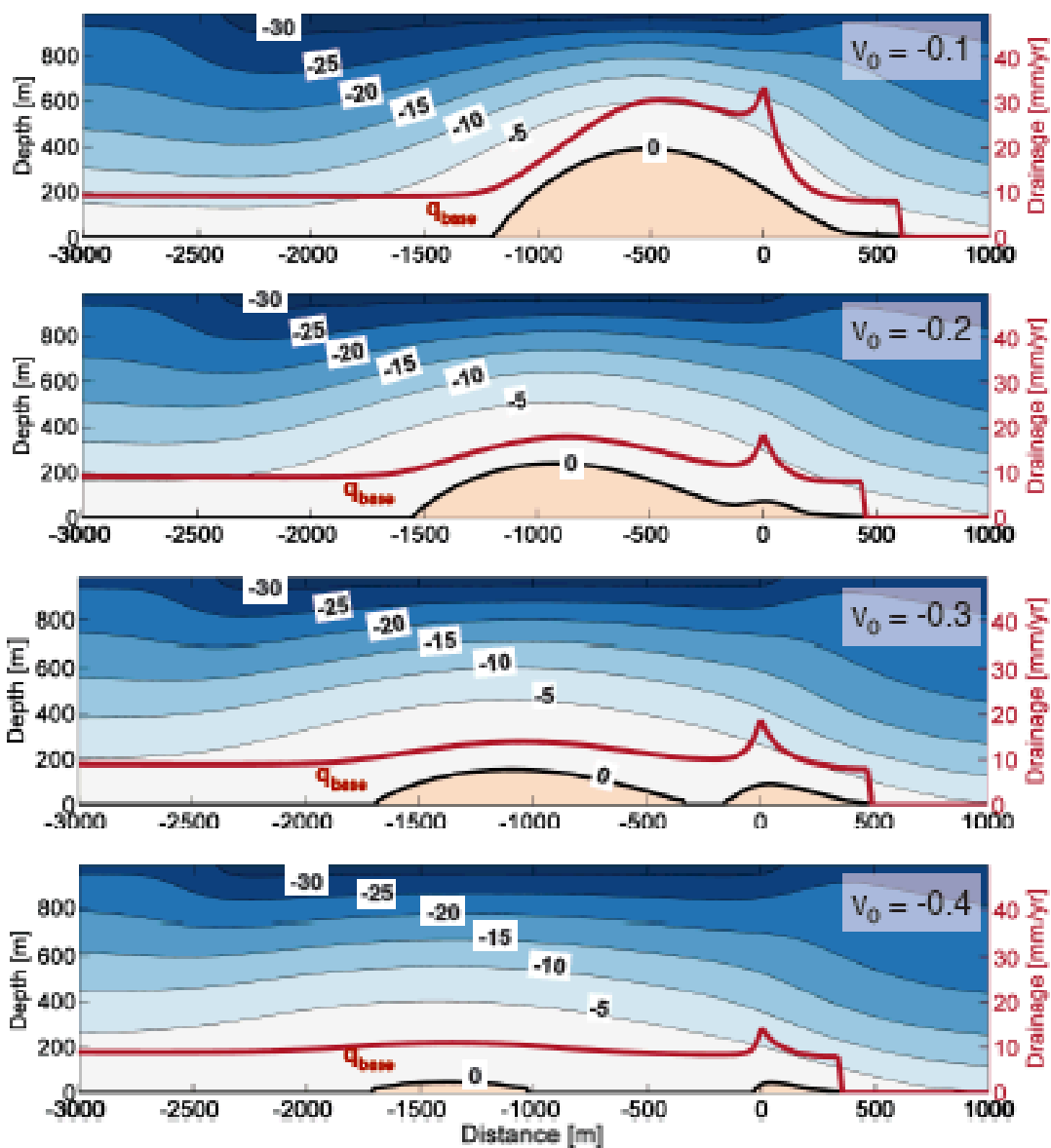
For $Q_w = 100\,\text{km} \times 41\,\text{m}^3/\text{m}\cdot\text{yr}$, and $S = 0.0012$:

Clarke sub-glacial flooding range

Assumed plausible here

	0.01	0.02	0.03	0.04
Manning Coefficient, n_M (s/m$^{1/3}$)	0.01	0.02	0.03	0.04
Equivalent Nikuradse Roughness, k (cm)	0.03	1.6	18.0	(101.1)
Channel Diameter, D (m)	0.9	1.1	1.3	1.5
Effective Normal Stress at Channel Margin, $\sigma_{hoop} - p_{ch}$ (kPa)	369	310	280	261

$$\sigma_{hoop} - p_{ch} = \frac{2}{3}(\sigma_o - p_{ch})$$

Sensitivity: $Q_w \to 0.25 Q_w \Rightarrow D \to 0.59 D$, $(\sigma_{hoop} - p_{ch}) \to 0.89(\sigma_{hoop} - p_{ch})$

σ_o

Ice (temperate near bed)

σ_o

σ_o

Water

D

Till (saturated)
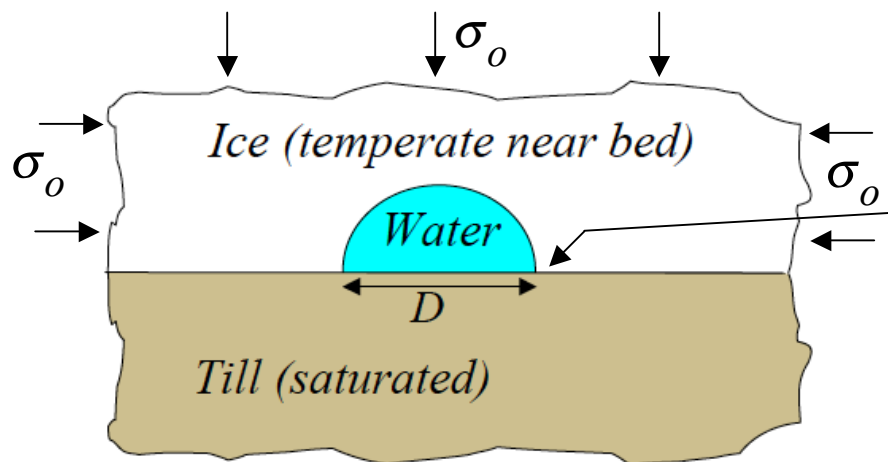
Strength $\tau_{ch} = f(\sigma_{hoop} - p_{ch})$

$\approx 0.5(\sigma_{hoop} - p_{ch}) \approx 150$ kPa

$\therefore \tau_{ch} / \tau_{base} \approx 20$ to 45

For the 6 major streams, τ_{ch} / τ_{base} average $= 32$, and range $= 12$ to 56.

• Meltwater channel formation is likely near a region of intense melt generation.

• A crack ending in a channel has greatly diminished stress and strain rate concentration, and the high Terzaghi effective stress just outside the channel will contribute to resisting continued expansion of the slipping zone.

$$\text{Strength ratio } \frac{\tau_{ch}}{\tau_{base}} = \frac{(\sigma_{hoop} - p_{ch})}{(\sigma_{base} - p_{base})} \approx 20 \text{ to } 45 \text{ (avg. 32)}$$

• **Very important: Advective effect** (not well yet modeled -- complicated!!) of slow lateral expansion of slipping zone towards the cold ice ridge could **chill ice near the stress concentrator** and **reduce or eliminate a partial-melt zone**.

Conjectured stable configuration:

Temperate ice zone
Melt-water channel, terminating slipping zone
locked; ice frozen to bed
slipping
ice sheet
sub-glacial till, gradually transitioning to bedrock
depressed pore pressure *p* means increased strength
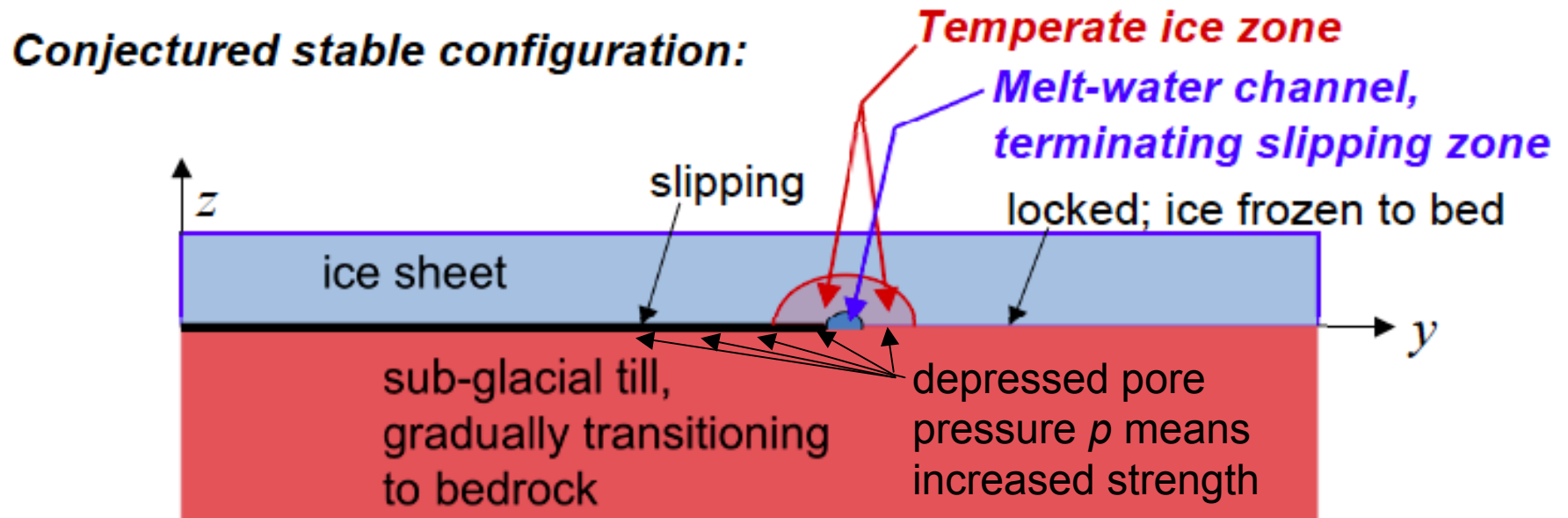
Field evidence, possible temperate ice and melt channels at margins

Borehole drilled into a dying shear margin of **Kamb** (**C**) ice stream

[Vogel (*PhD Thesis*, Caltech, 2004) and Vogel et al. (*Geophys Res Lett*, 2005)]
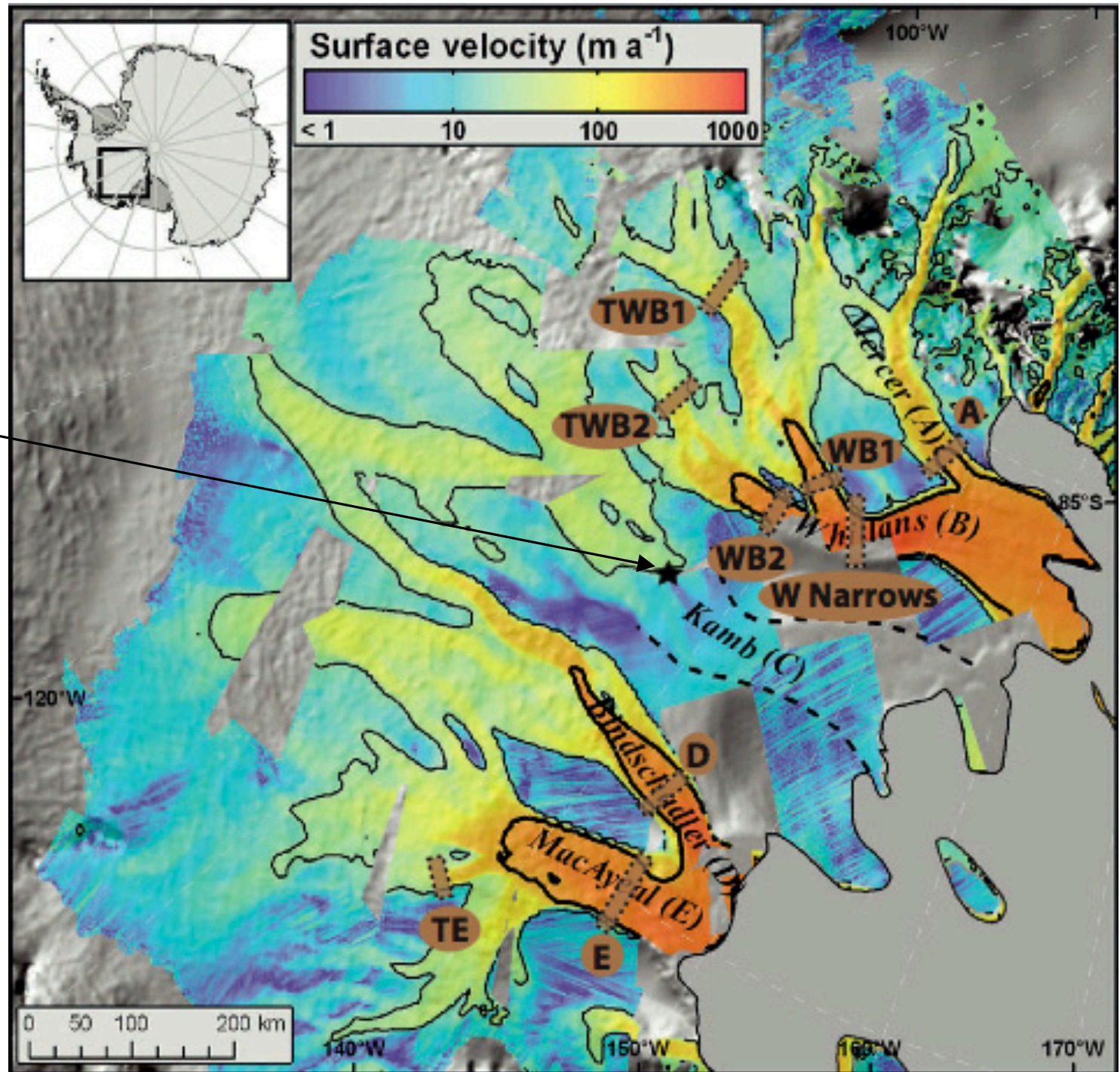
Evidence of channel at margins
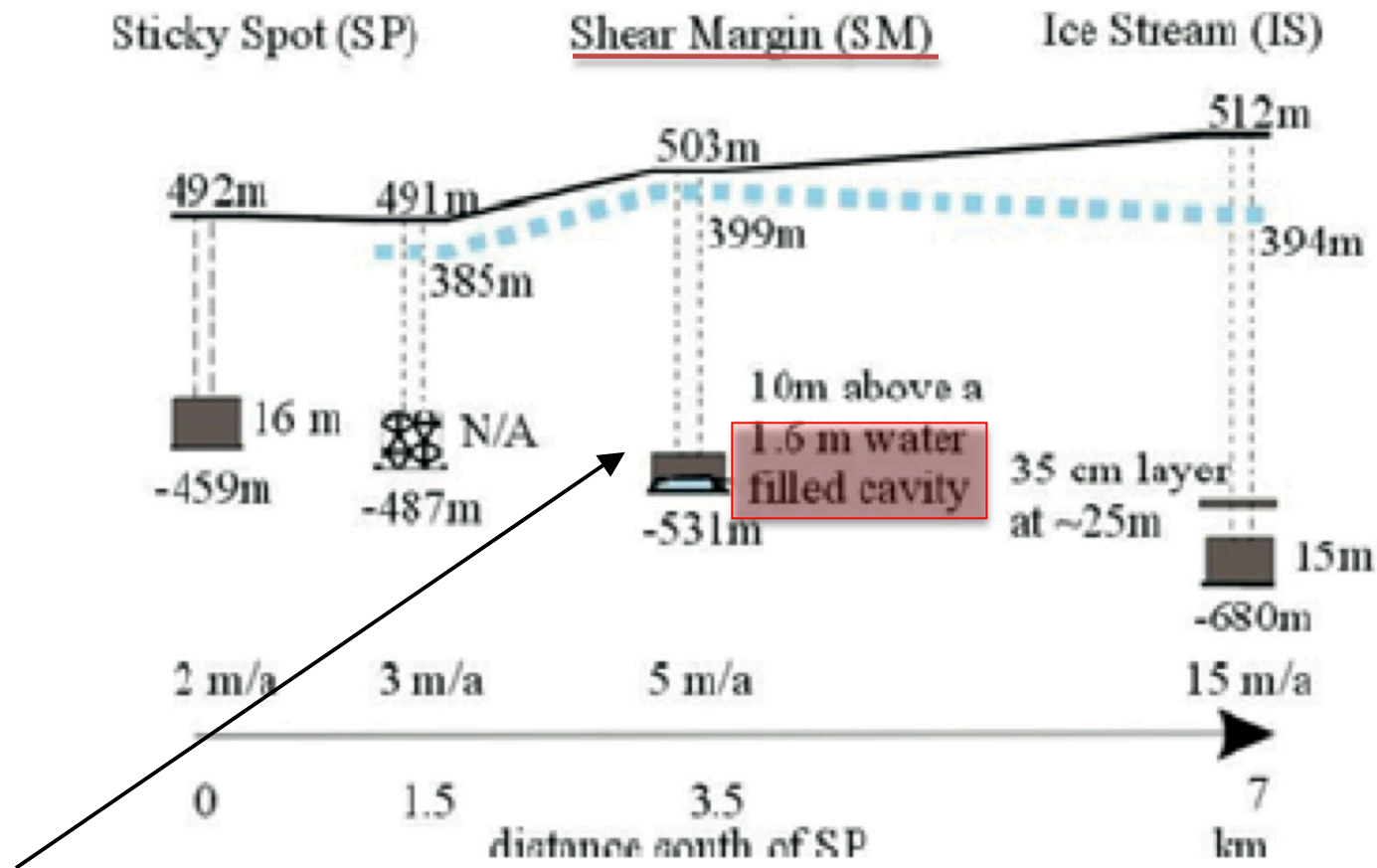
Borehole observation at the presently inactive shear margin of **Kamb** (**C**) ice stream:
• Found a 1.6 m tall water-filled cavity between ice sheet and bed.
• Video of the borehole shows horizontal acceleration of particles sinking into the cavity, indicating flow of water within the cavity -- part of a channel?
[Modified from Vogel PhD (Thesis, 2004) and Vogel et al. (GRL, 2005).]

Possible field evidence of internal melting at margins
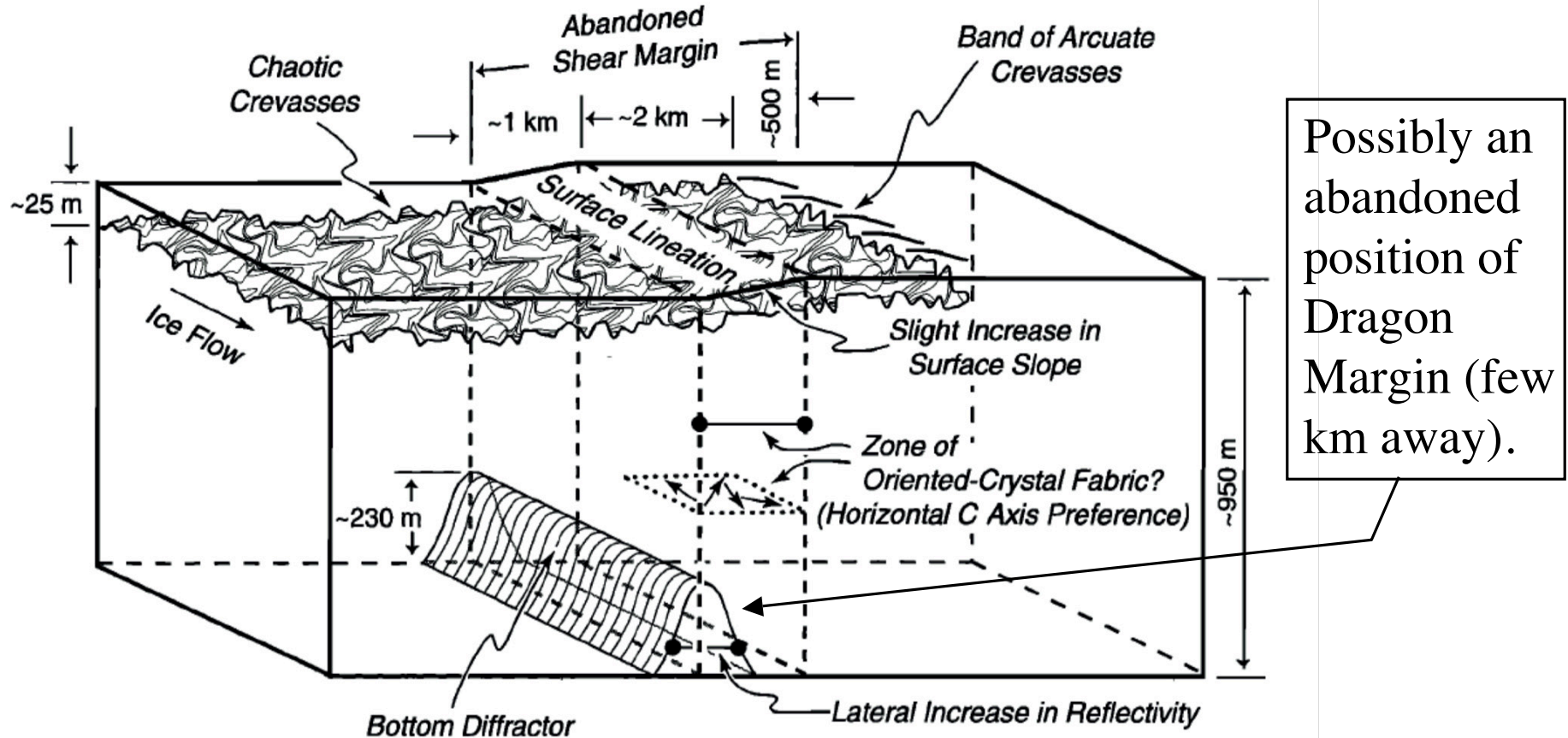
• Clarke et al. [2000], in order to explain the bottom diffractors, have invoked **partial melting in temperate ice to a height of 230 m**, due to strain heating, among other possibilities (**entrained sediments**, **bottom crevasses**).
• Also, Clarke et al. noted a personal communication from H. Engelhardt (Caltech): Abnormal drill resistance encountered from ≈ 56 m above bed. Fresh scratches found on drill tip (assumed to due to **entrained sediments**).

Flow of ice around a channel

- A finite radius of curvature at a crack tip blunts the stress concentration.

- Motivated by this we consider a slipping to locked transition that occurs across a channel.

Downstream ice velocity, m/yr

Newtonian ice, channel radius R = 1m

Newtonian vs. Glen's law ice, channel radius R = 1m

Newtonian

Glen's law

Shear stress on bed, kPa

Distance from margin, m
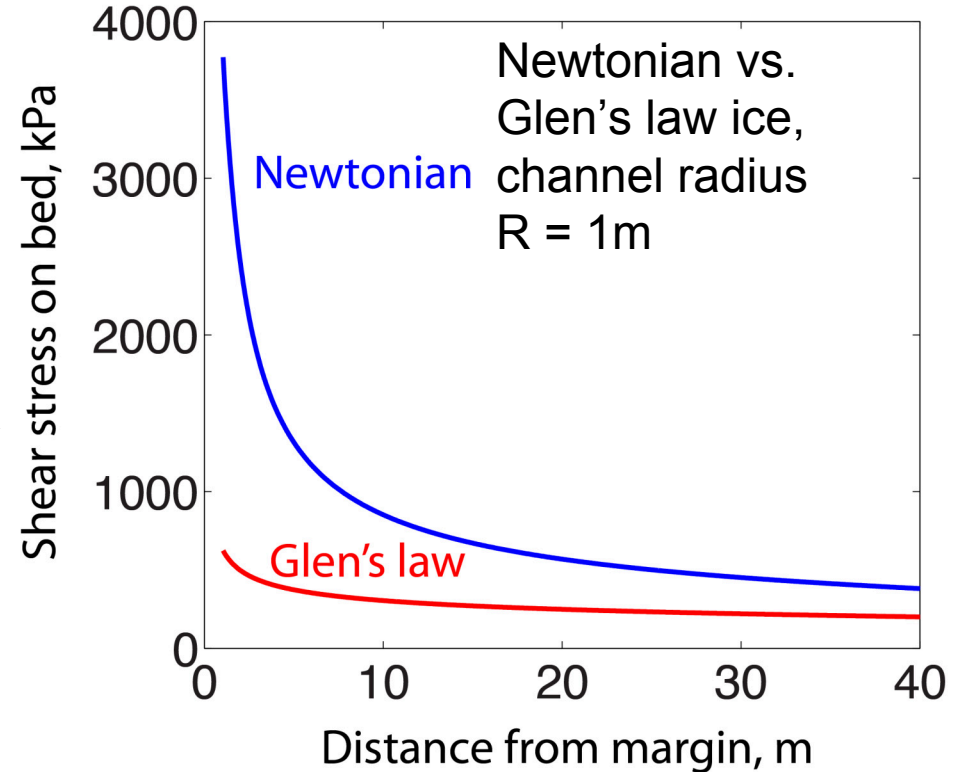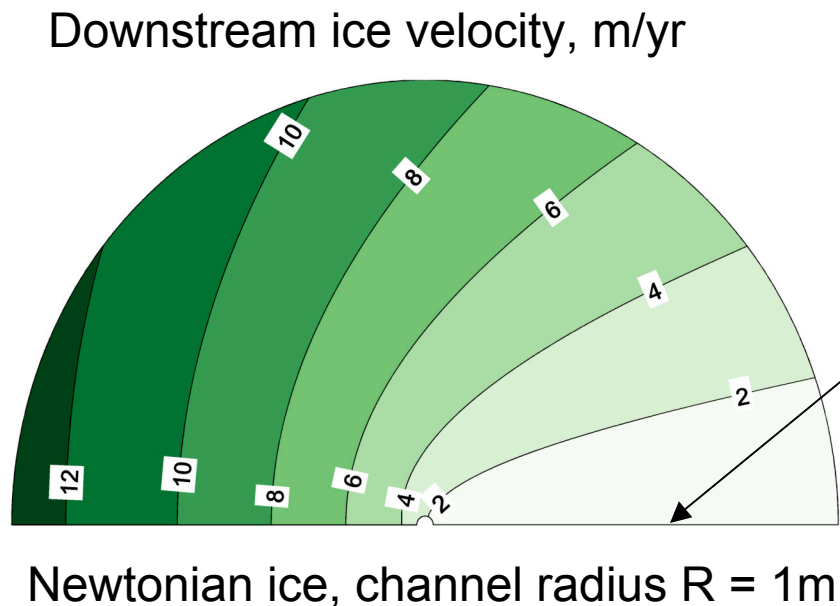
Influence of channel size

- The channel size also influences the maximum stress on the bed.

Newtonian

Glen's law
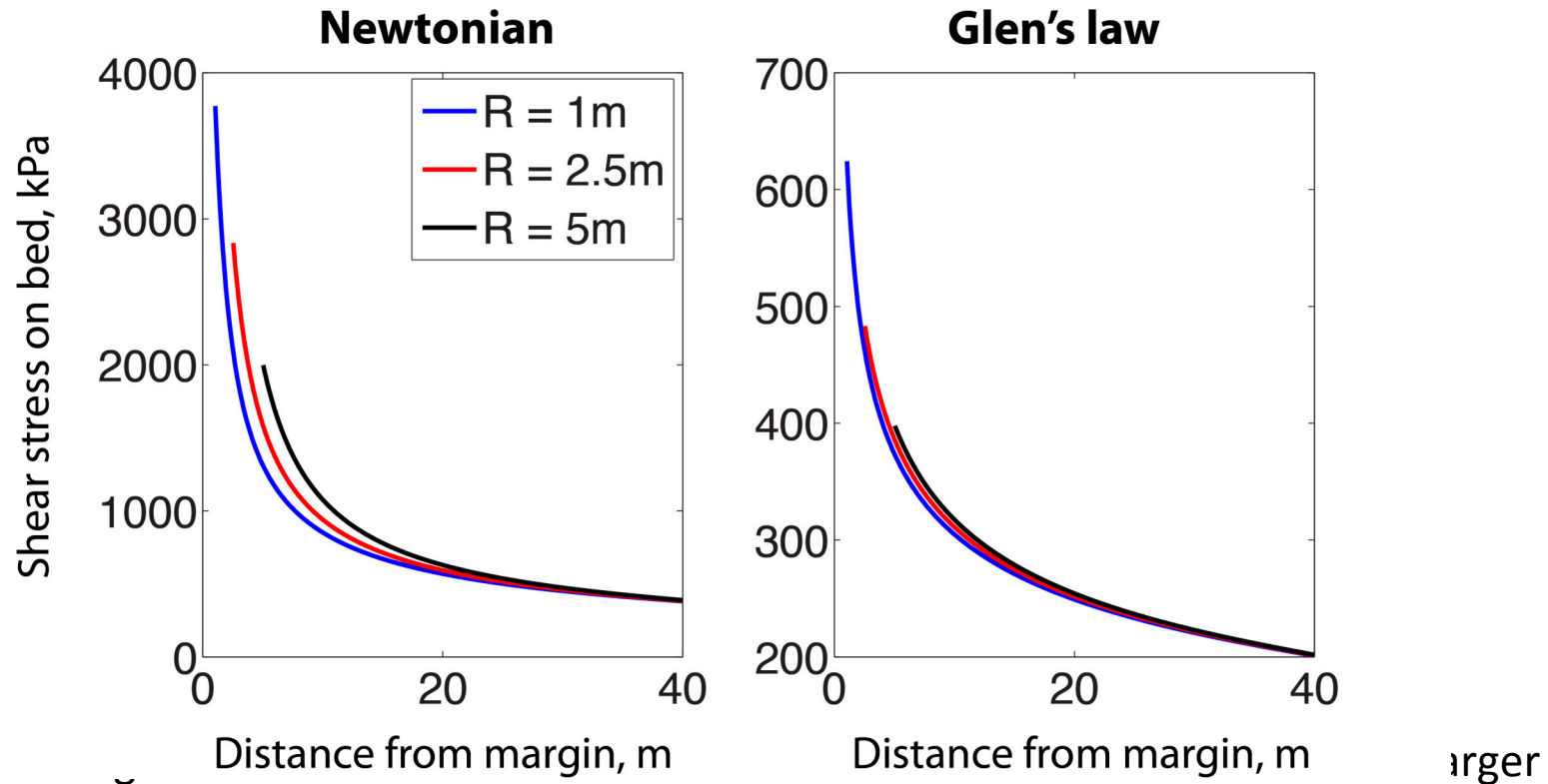
- ⎯ ⎯ ⎯ Distance from margin, m ⎯ ⎯ ⎯ Distance from margin, m ⎯ ⎯ larger strength of the ice-bed interface.

- Even if the the stress is larger than the strength the channel may still facilitate locking through a cohesive zone (Dugdale [1960]; Barenblatt [1962]; Schoof [2012]).

Conclusions, West Antarctic Ice Stream Margins

- The transition from a slipping to a locked bed concentrates stress beneath the ridge.

- Our model (**without lateral advection**) predicts that shear heating at the margin leads to temperate ice and melting for almost all ice streams. This melt may lead to channel formation beneath ice stream margins.

- Such a channel limits the maximum stress on the locked bed and increases the strength of the ice-bed interface, providing a mechanism that may facilitate locking.

- The maximum stress on the locked portion of the bed decreases with increasing channel radius.

- Glen's law leads to a substantially lower maximum stress than a Newtonian rheology.

- We have **not proven** that the mechanisms outlined could stabilize a shear margin, and we **have no simple way at present of more fully evaluating the important effects of lateral thermal advection**. So this is an **interim report**!

Shear localization due to thermal pressurization of pore fluids in rapidly sheared granular media (rapidly sheared fault zones)

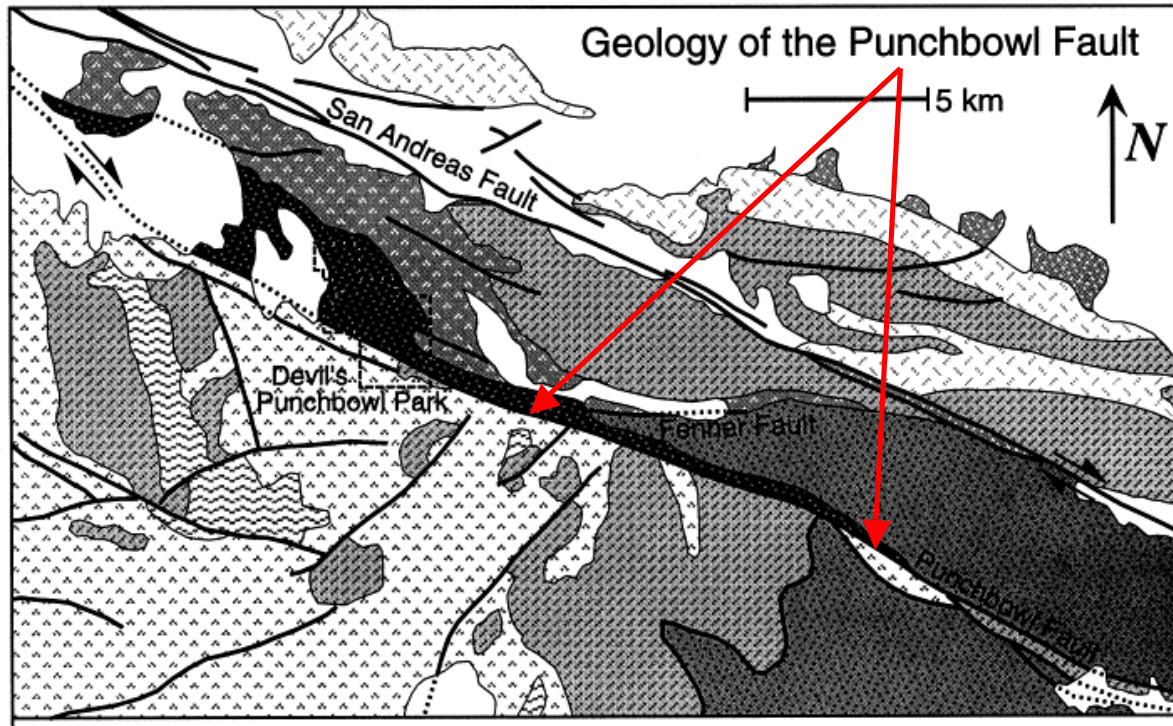**Nicolas Brantut** (Univ. Col. London)
John D. Platt (Harvard)
James R. Rice (Harvard)
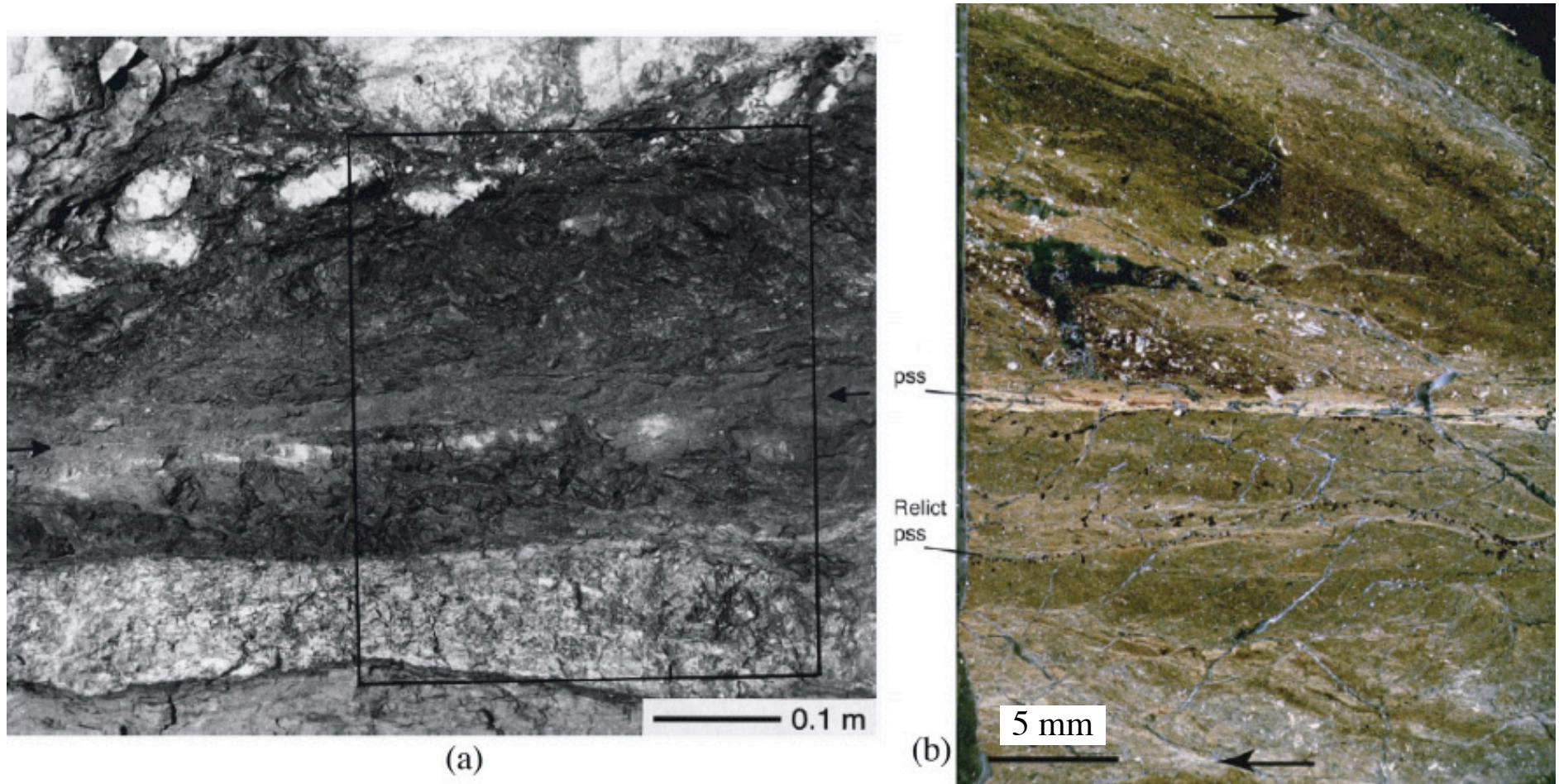**John W. Rudnicki** (Northwestern)

Geology of the Punchbowl Fault

Chester & Chester [*Tectonophys.*, 1998]
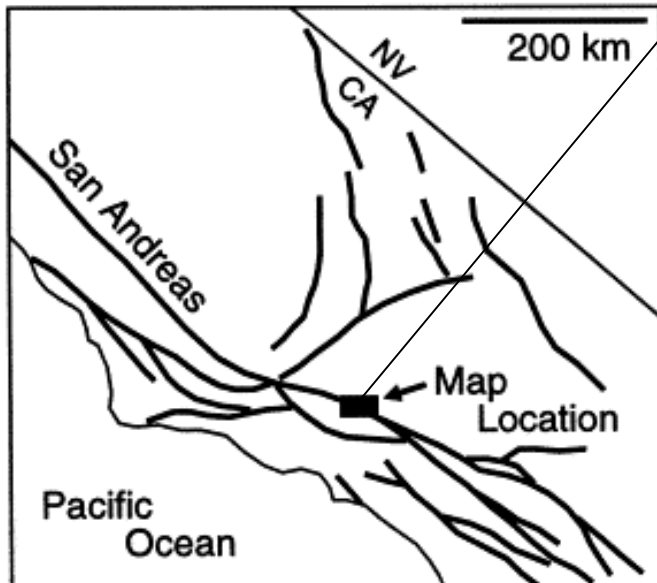
Earthquake shear is highly localized

Punchbowl PSS, composite based on Chester & Chester [*Tectonophys* '98] & Chester & Goldsby [*SCEC* '03]

Figure 1. Principal slip surface (PSS) along the Punchbowl fault. (a) From *Chester and Chester* [1998]: Ultracataclasite zone with PSS marked by black arrows; note 100 mm scale bar. (b) From Chester et al. (manuscript in preparation, 2005) [also *Chester et al.*, 2003; *Chester and Goldsby*, 2003]: Thin section; note 5 mm scale bar and ~1 mm localization zone (bright strip when viewed in crossed polarizers due to preferred orientation), with microshear localization of most intense straining to ~100−300 μm thickness.
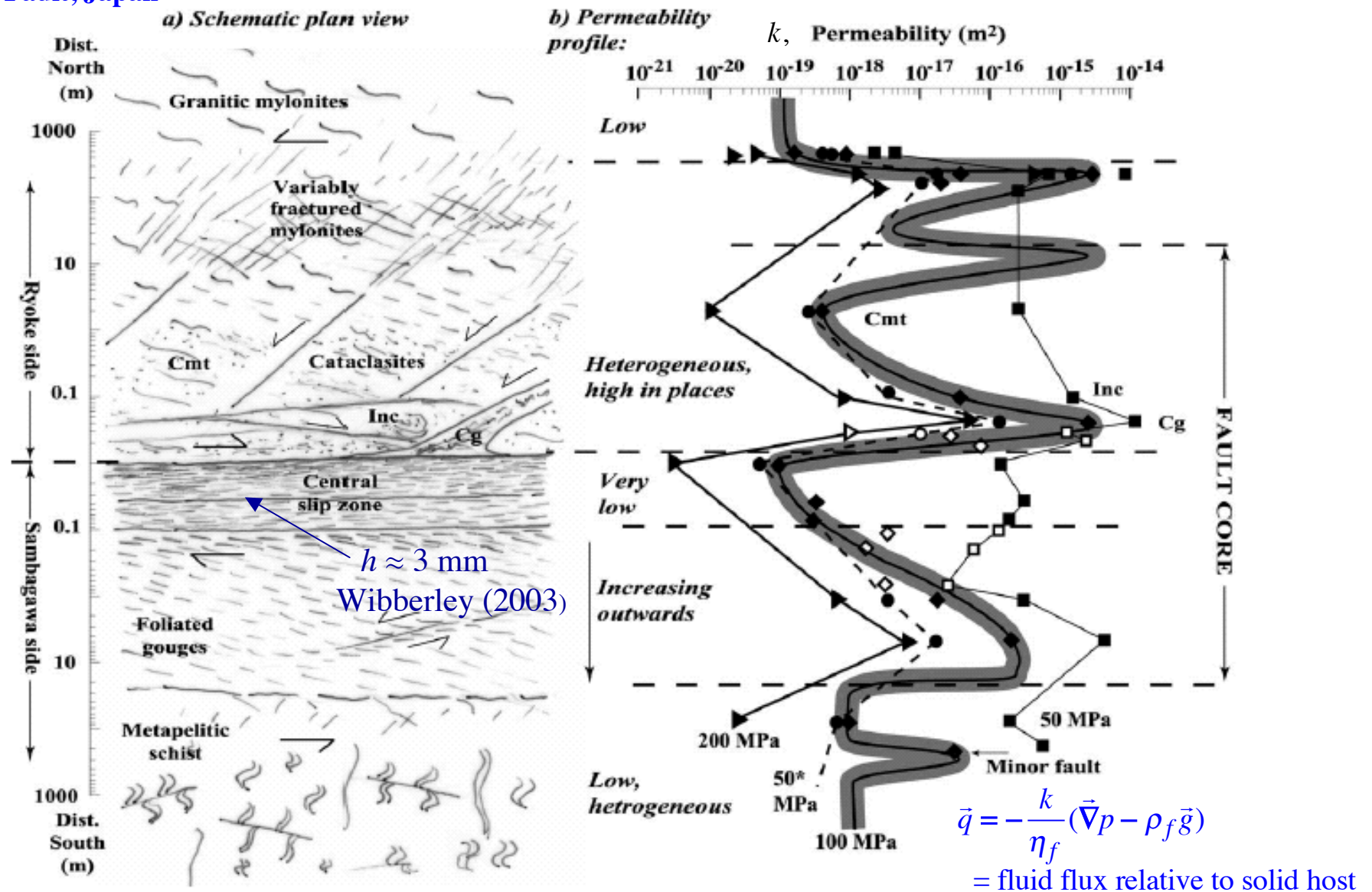
Fig. 11. Sketch summary of the main elements of permeability structure across the Median Tectonic Line. (a) Summary of the structural zones; (b) summary permeability data distribution for different confining pressures (stated at the base, with * denoting data from the deconfining path), for 20 MPa pore pressure, given the mapped distribution of fault rocks shown in Figs. 1–3. Note that the distance axis is logarithmic in both directions away from the Ryoke/Sambagawa contact. 'Cmt' and 'Inc' denote cemented and incohesive foliated cataclasites, respectively, and 'Cg' denotes crenulated gouge.

$$\tau = f \times (\sigma_n - p)$$

Statically strong but *dynamically weak faults, e.g., due to thermal weakening in rapid, large slip:*

- • *Process expected to be important from start of seismic slip:*
- **- Thermal pressurization of in-situ pore fluid, reduces effective stress.**

- • *Process that may set in at large enough rise in T:*
- **- Thermal decomposition, fluid product phase at high pressure** (*e.g.*, CO_2 **from carbonates;** H_2O **from clays or serpentines).**

- • *Ultimately:*
- **- Melting at large slip,** *if* **above have not limited increase of** *T*.

Shear of a fluid-saturated gouge layer
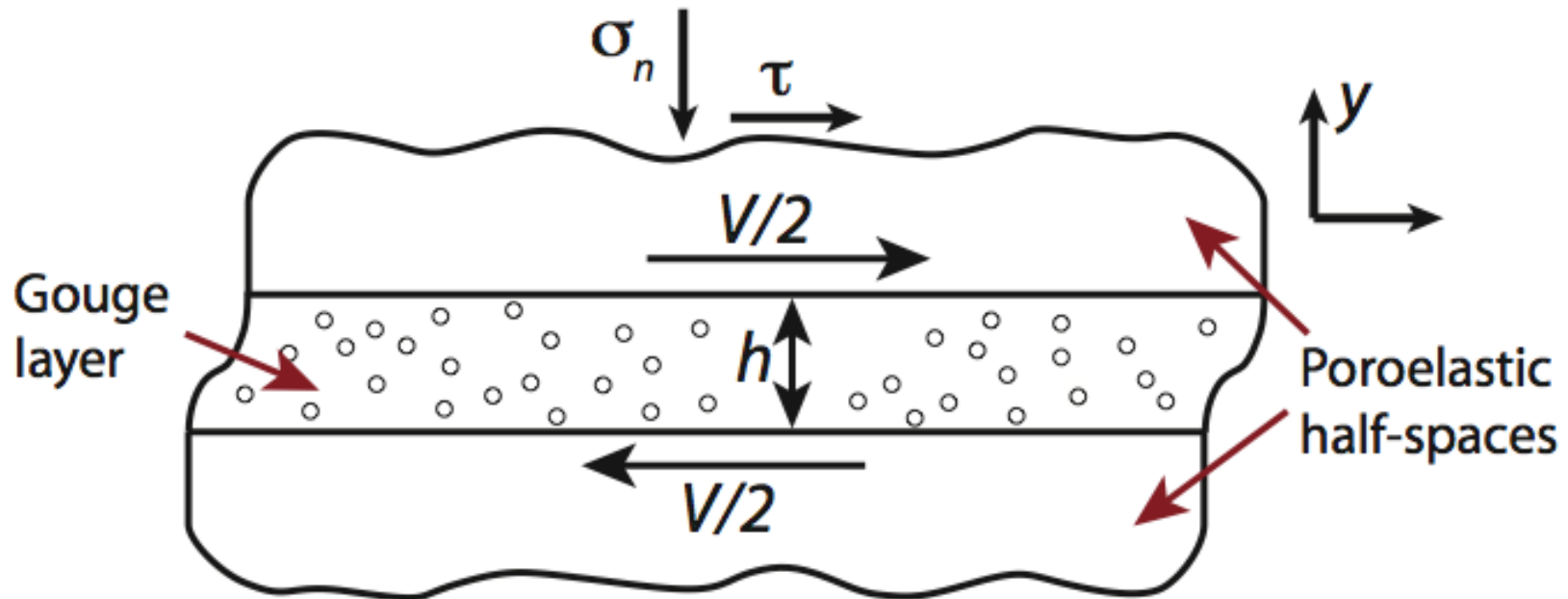
- Two non-yielding half-spaces are moved relative to each other at a speed *V.*

- All inelastic deformation accommodated in gouge layer, leading to a nominal strain rate $\dot{\gamma}_0 = V / h$.

Thermo-mechanical model in gouge layer

• To model the deforming gouge layer we use,

Mechanical equilibrium

$$\frac{\partial \tau}{\partial y} = 0, \quad \frac{\partial \sigma_n}{\partial y} = 0$$

Conservation of energy

$$\frac{\partial T}{\partial t} - \alpha_{th} \frac{\partial^2 T}{\partial y^2} = \frac{\tau \dot{\gamma}}{\rho c}$$

Conservation of fluid mass

$$\frac{\partial p}{\partial t} - \alpha_{hy} \frac{\partial^2 p}{\partial y^2} = \Lambda \frac{\partial T}{\partial t}$$

• Shear stress modeled using the effective stress and **rate-strengthening** friction,

$$\tau = f(\dot{\gamma})(\sigma_n - p) \qquad f(\dot{\gamma}) = f_0 + (a - b)\log\left(\frac{\dot{\gamma}}{\dot{\gamma}_0}\right)$$

$$\left(We \text{ assume } a - b \equiv \left(\dot{\gamma} df(\dot{\gamma}) / d\dot{\gamma} \right)_{\dot{\gamma} = \dot{\gamma}_0} > 0 \right)$$

Shear between moving rigid blocks of perfectly insulating, impermeable material :

Exact homogeneous shear solution (Lachenbruch, JGR, 1980, version ignoring dilatancy) :

$$\tau(t) = f_o\big(\sigma_n - p(t)\big) = f_o\big(\sigma_n - p_a\big)\exp\left(-f_o\frac{\Lambda}{\rho c}\dot{\gamma}_o t\right),$$

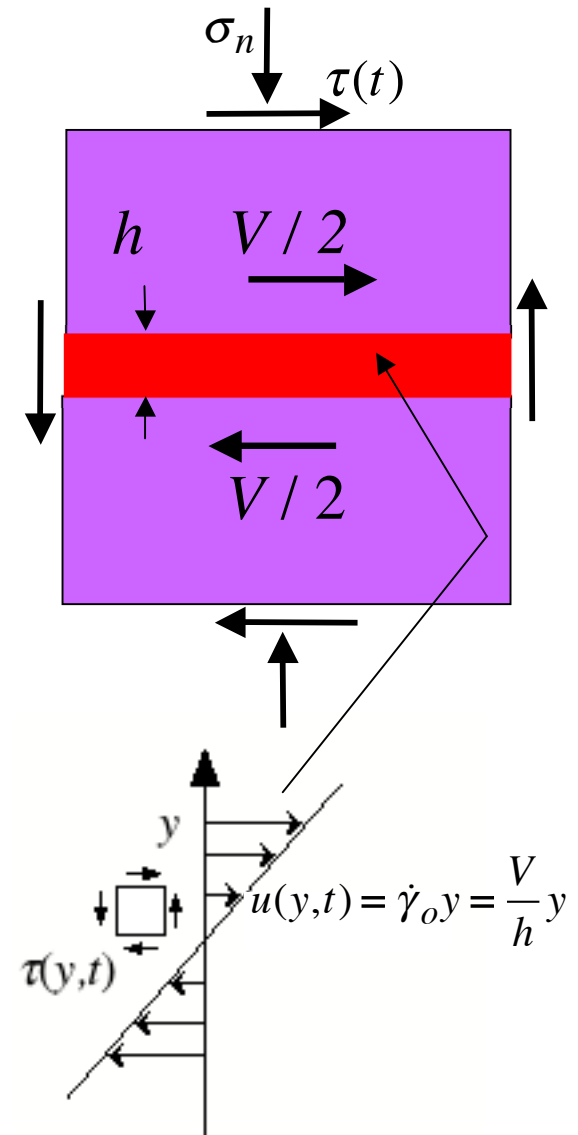$$\big(\dot{\gamma}_o = V/h\big)$$

Linearized perturbation analysis :

Is that solution stable to small perturbations?
Not unless h is very small !

Stable only if $h \le W_{crit} \equiv \dfrac{\pi^2}{\left(2+\dfrac{f_o}{a-b}\right)}\dfrac{\rho c}{f_o \Lambda}\dfrac{\big(\alpha_{th}+\alpha_{hy}\big)}{V}.$

$$\left(\text{Typically, }\frac{f_o}{a-b}\gg 2 \ \Rightarrow\ W_{crit}\approx \pi^2\frac{a-b}{f_o^2}\frac{\rho c}{\Lambda}\frac{\alpha_{th}+\alpha_{hy}}{V}\ .\right)$$

[$W_{crit} = \lambda_{shr}/2$, where λ_{shr} = longest wavelenth λ for stable linearized response to infinitesimal $\exp(2\pi i y/\lambda)$ perturbation]

σ_n

$\tau(t)$

h $V/2$

$V/2$

y

$u(y,t) = \dot{\gamma}_o y = \dfrac{V}{h}y$

$\tau(y,t)$

Rice, Rudnicki & Platt (to JGR 2013)

Estimates of maximum stable shear layer thickness (i.e., localized zone width) W_{crit}

$Results$, using $f_o = 0.4$, $\dfrac{f_o}{a-b} = 20$, $V = 1 \dfrac{m}{s}$, $\alpha_{th} = 0.7 \dfrac{mm^2}{s}$, $\rho c = 2.7 \dfrac{MPa}{°C}$:

Corresponding to ~ 7 km depth :

Low estimate (Based on lab properties of ***intact*** Median Tectonic Line gouge [Wibberley and Shimamoto, 2003] at effective confining stress = 125 MPa and T = 200°C):

$$\Lambda = 0.70 \frac{MPa}{°C} , \quad \text{and} \quad \alpha_{hy} = 1.5 \frac{mm^2}{s} \quad \Rightarrow \quad \boxed{W_{crit} = \text{3-5 μm}}$$

High estimate (Accounts very roughly for ***fresh damage*** of the initially intact fault gouge, introduced at the rupture front just before and during shear, by increasing permeability k to k^{dmg} = 5-10 k, and increasing drained compressibility β_d to β_d^{dmg} = 1.5-2 β_d):

$$\Lambda \approx 0.34 \frac{MPa}{°C} , \quad \text{and} \quad \alpha_{hy} \approx 3.5 \frac{mm^2}{s} \quad \Rightarrow \quad \boxed{W_{crit} \approx \text{25-40 μm}}$$

Corresponding to ~ 1 km depth :

Low estimate : $\boxed{W_{crit} \sim \text{25 μm}}$ *High estimate :* $\boxed{W_{crit} \sim \text{200 μm}}$

Rice, Rudnicki & Platt (to JGR 2013)

Shear of a fluid-saturated gouge layer:
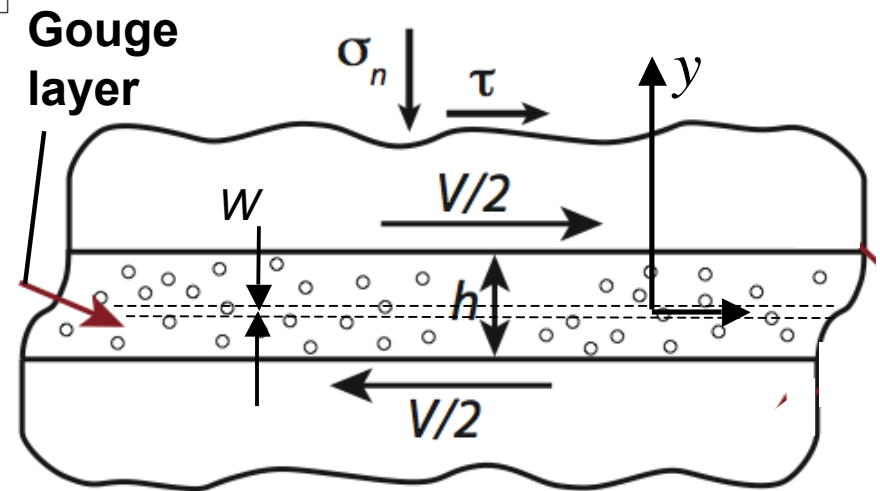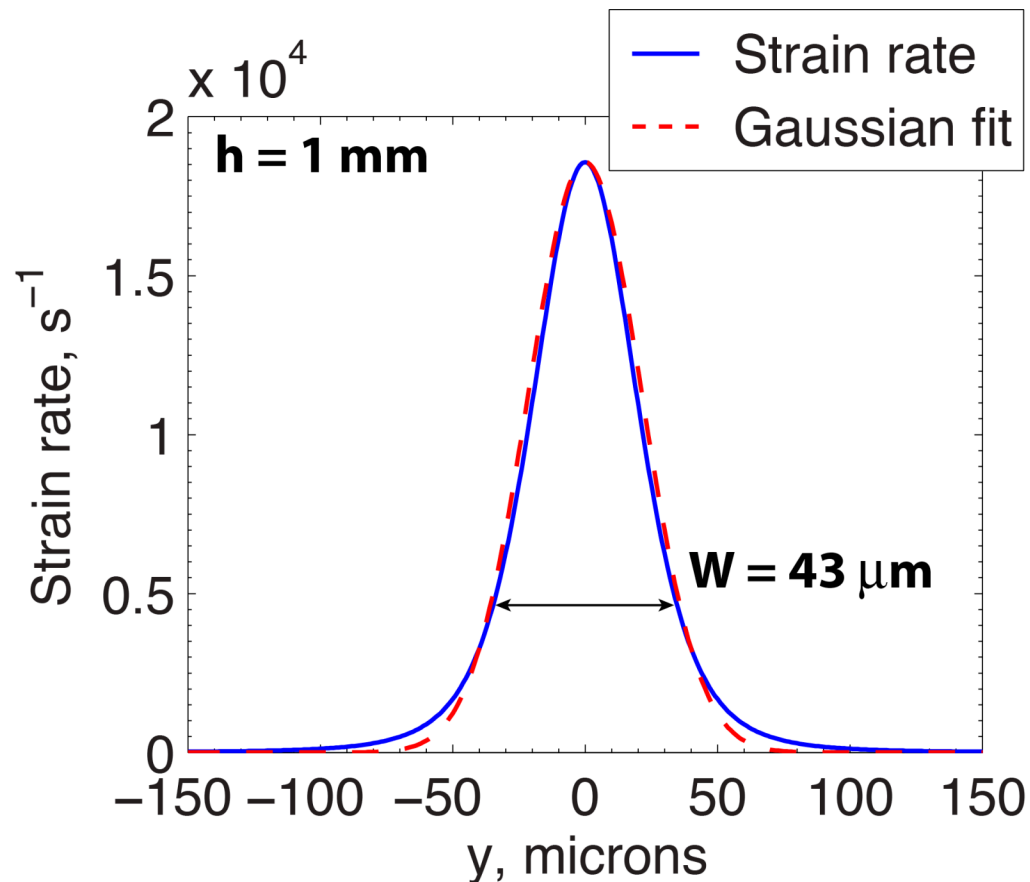**Full nonlinear numerical solutions**
(vs. linearized perturbations)
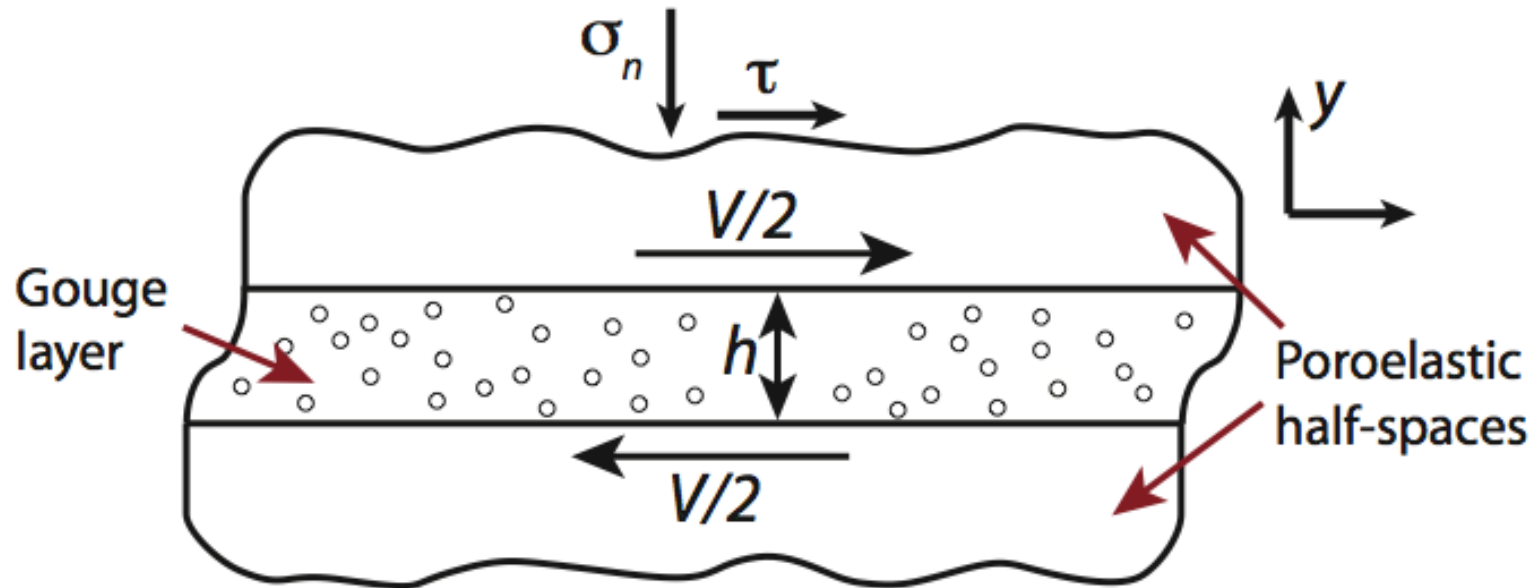
• Two non-yielding half-spaces are moved relative to each other at a speed *V* (taken as 1 m/s).

• All deformation accommodated in gouge layer leading to a nominal strain rate, $\dot{\gamma}_o = \dfrac{V}{h}$

Strain localization

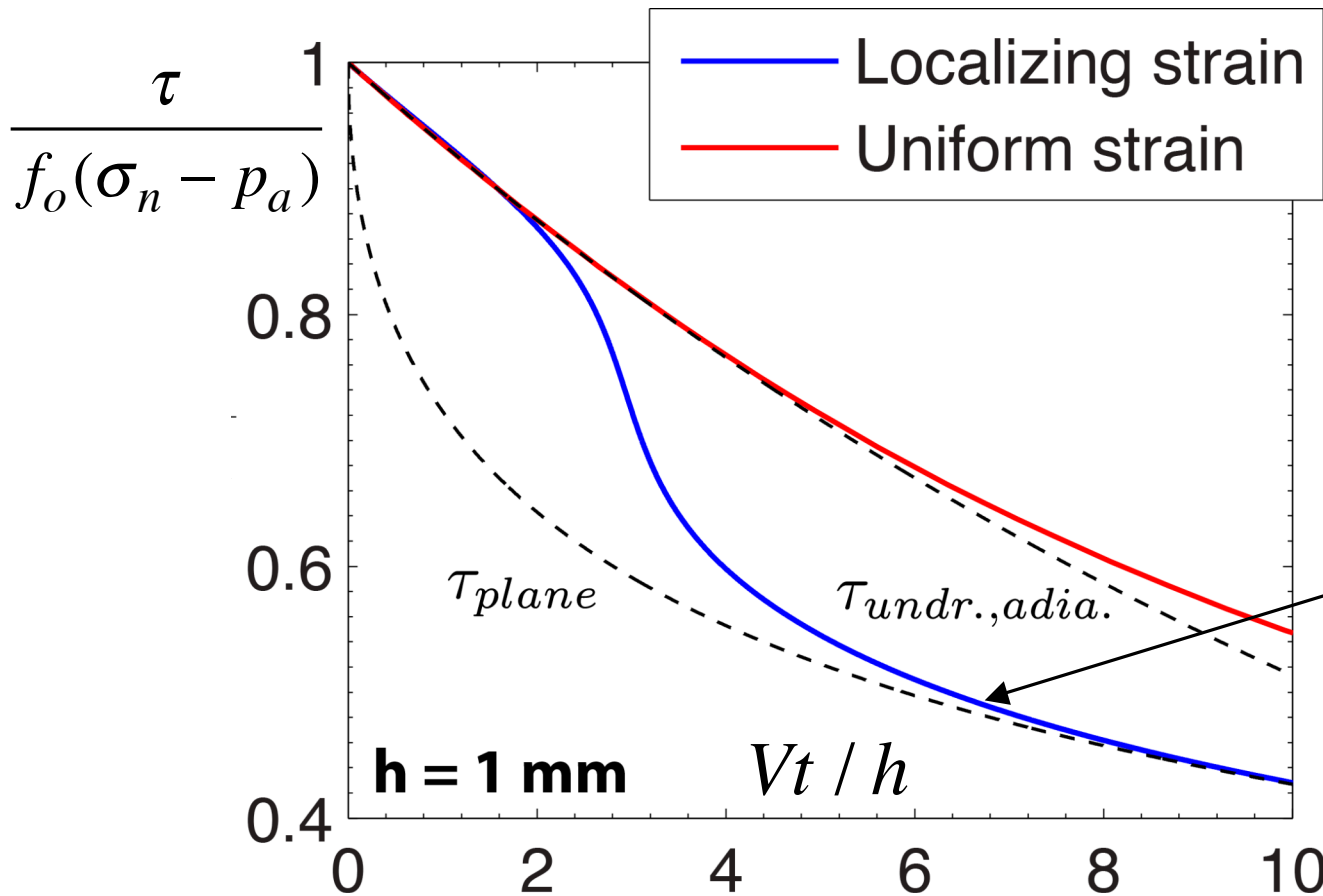• Simulations using representative physical values show that strain localization does occur.

• Deformation has localized to a zone much thinner than the layer, $W << h$ (43 μm << 1,000 μm).
$W_{\text{nonlin. calc.}}$ is comparable to $W_{\text{lin. pert.}}$.

Implications, dynamic weakening

• Weakening of the gouge layers during localization

• Localization leads to additional weakening.

$$\tau = f_o(\sigma_n - p_a)\exp\left(\frac{Vt}{L^*}\right)\mathrm{erfc}\left(\sqrt{\frac{Vt}{L^*}}\right) \quad \text{where} \quad L^* = \left(\frac{2\rho c}{f_o \Lambda}\right)^2 \frac{\left(\sqrt{\alpha_{hy}} + \sqrt{\alpha_{th}}\right)^2}{V}$$

$$\tau = f \times (\sigma_n - p)$$

Statically strong but *dynamically weak* *faults, e.g., due to thermal weakening in rapid, large slip:*

  • *Process expected to be important from start of seismic slip:*
  - **Thermal pressurization** **of in-situ pore fluid,**
      reduces effective stress.

 • *Process that may set in at large enough rise in T*:
  - **Thermal decomposition, fluid product phase at high pressure**
      (*e.g.,* **CO$_2$ from carbonates; H$_2$O from clays or serpentines).**

 • *Ultimately:*
  - **Melting at large slip,** *if* **above have not limited increase of** *T*.

Examples, thermal decomposition:

Dolomite, **CaMg(CO$_3$)$_2$** (De Paola et al., *Tectonics*, 2008, *Geology*, 2011; Goren et al., *J. Geophys. Res.,* 2010):

- At $T \sim 550°C$, dolomite decomposes to calcite, periclase, and carbon dioxide:
$$CaMg(CO_3)_2 \rightarrow CaCO_3 + MgO + CO_2$$
- At $T \sim 700$-$900°C$, the calcite further decomposes to lime and carbon dioxide:
$$CaCO_3 \rightarrow CaO + CO_2$$

Many clays and hydrous silicates (Brantut et al., *J. Geophys. Res.,* 2010):

- At $T \sim 500°C$ ($\sim 300°C$ for smectite, $\sim 800°C$ for chlorite), decomposition releasing H_2O starts.

Gypsum, **CaSO$_4$(H$_2$O)$_2$)** (Brantut et al., *Geology*, 2010):

- At $T \sim 100°C$, gypsum dehydrates to form bassanite:
$$CaSO_4(H_2O)_2 \rightarrow CaSO_4(H_2O)_{0.5} + 1.5\ H_2O$$
- At $T \sim 140°C$, bassanite turns into anhydrite
$$CaS_4(H_2O)0.5 \rightarrow CaSO_4 + 0.5\ H_2O$$

[Han, Shimamoto, Hirose, Ree & Ando, *Sci.,* 2007]:

Carbonate Faults

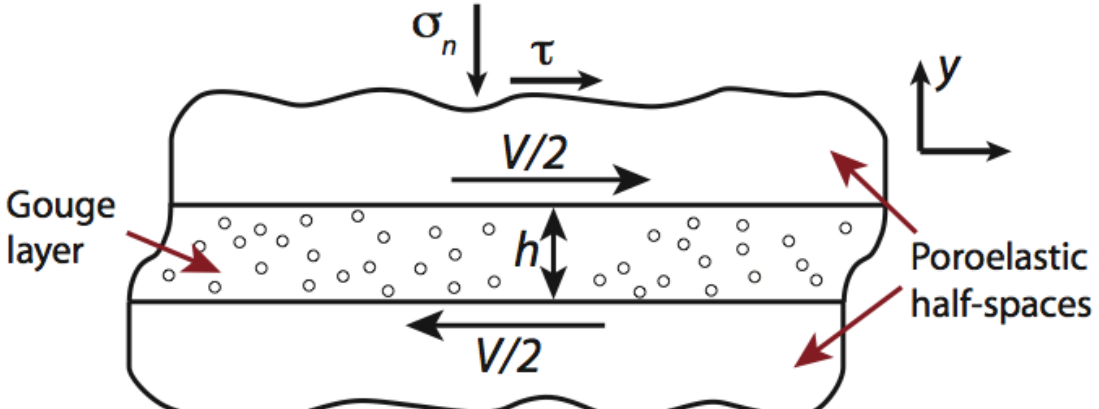Simulated faults in Carrara Marble at subseismic to seismic slip rates)
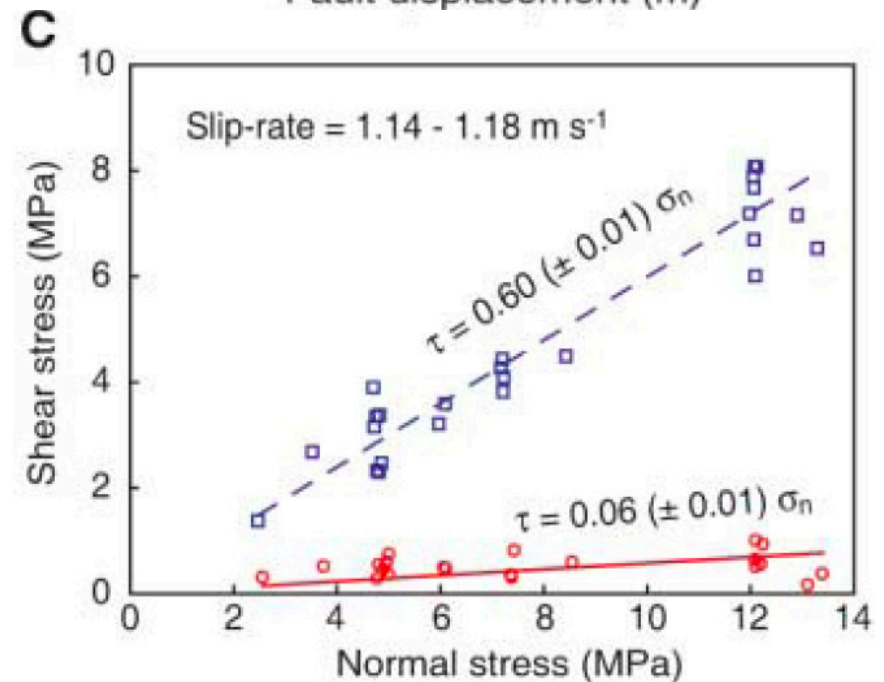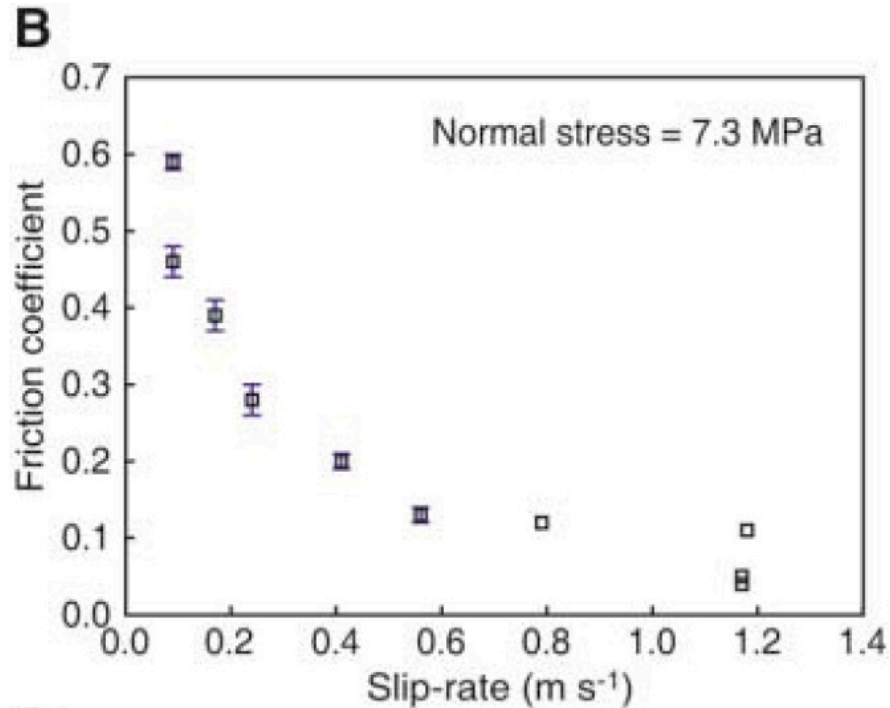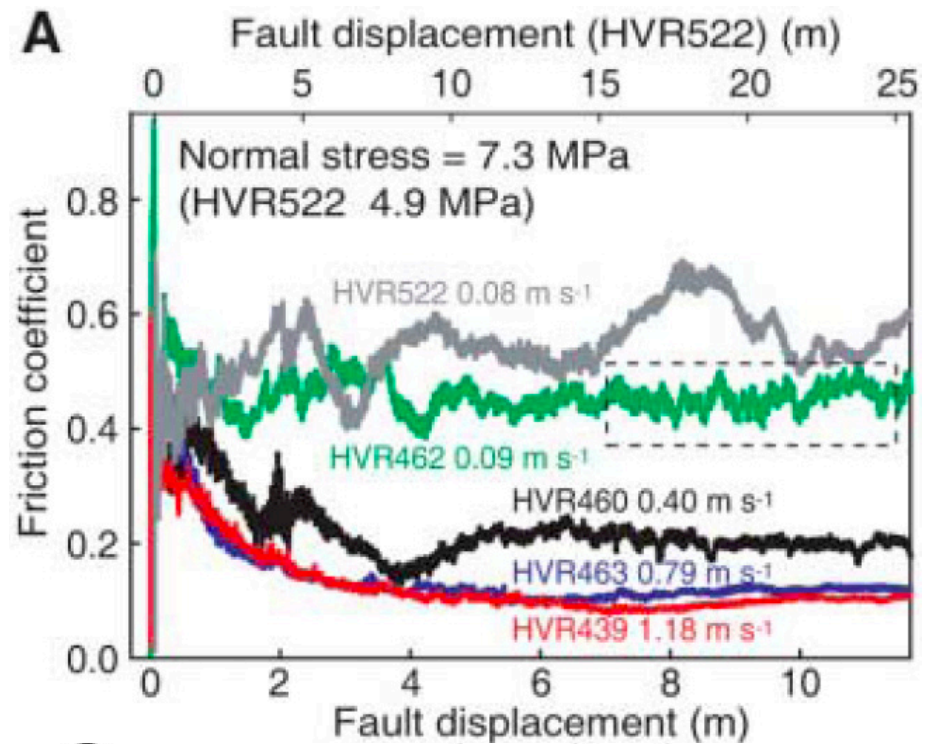
Model for decomposing gouge material

- To model the deforming gouge layer we use, based on J. Sulem and co-workers (Vardoulakis, Brantut, Ghabezloo, Famin, Lazar, Noda, Schubnel, Stefanou, Veveakis, …),

$$\frac{\partial T}{\partial t} = \frac{\tau\dot{\gamma}}{\rho c} + \alpha_{th}\frac{\partial^2 T}{\partial y^2} - E_r\frac{\partial \xi}{\partial t}$$

$$\frac{\partial p}{\partial t} = \Lambda\frac{\partial T}{\partial t} + \alpha_{hy}\frac{\partial^2 p}{\partial y^2} + P_r\frac{\partial \xi}{\partial t}$$

- We assume that the reaction follows an Arrhenius kinetic law,

$$\frac{\partial \xi}{\partial t} = A(1-\xi)\exp\left(-\frac{Q}{RT}\right)$$

Platt, Brantut & Rice (AGU, Fall 2011; in prep 2013 for *JGR*)

Different materials

Mineral	E_r (K)	P_r (MPa)	W (μm)	T_c (°C)
Calcite	3093	6421	18	900
Lizardite	228	1252	6	550
Kaolinite	3627	1599	86	500
Talc	496	586	32	800
Gypsum	89	1635	2	100

Linear perturbation estimate, localization zone width W, high T, fast reaction rate limit:

$$W = \pi^2 \frac{\alpha_{hy}}{V} \frac{(a-b)}{f_o^2} \frac{\rho c E_r}{P_r}$$

Representative simulations: thermal pressurization of in-situ fluids, followed by thermal decomposition
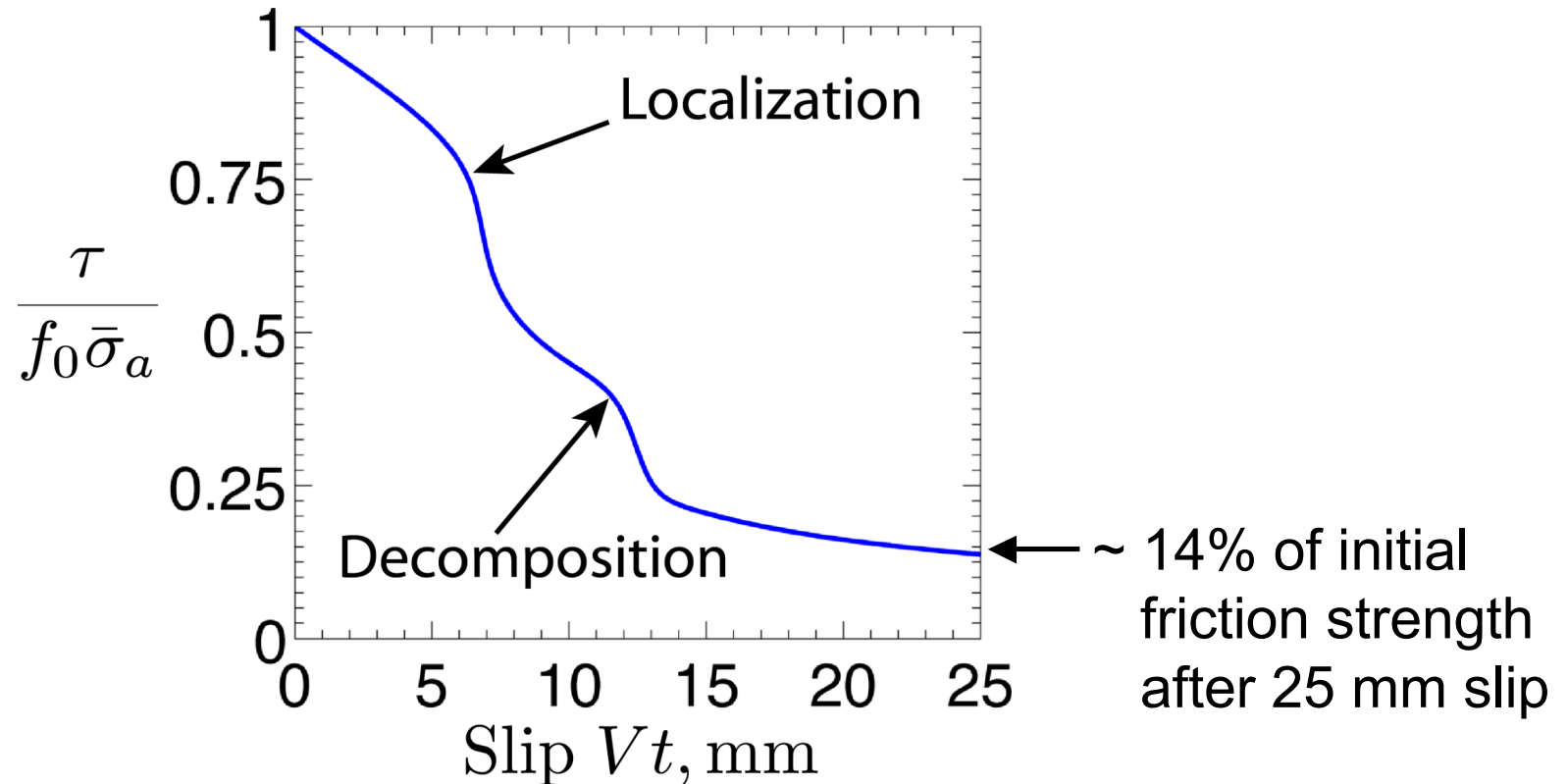
Suggests faults may be *strong* but *brittle* (quickly lose strength after slip is initiated at a place of localized stress concentration)
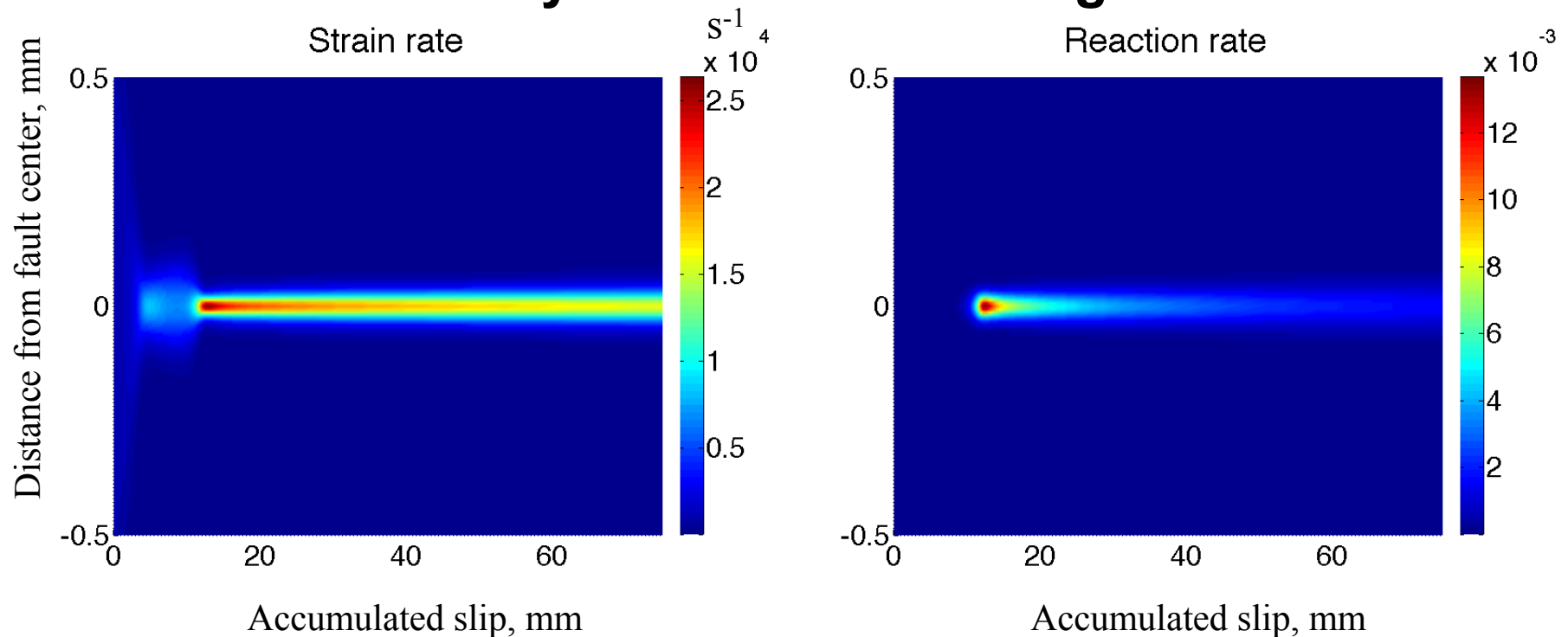
- Numerical solutions for a 1 mm wide gouge layer.
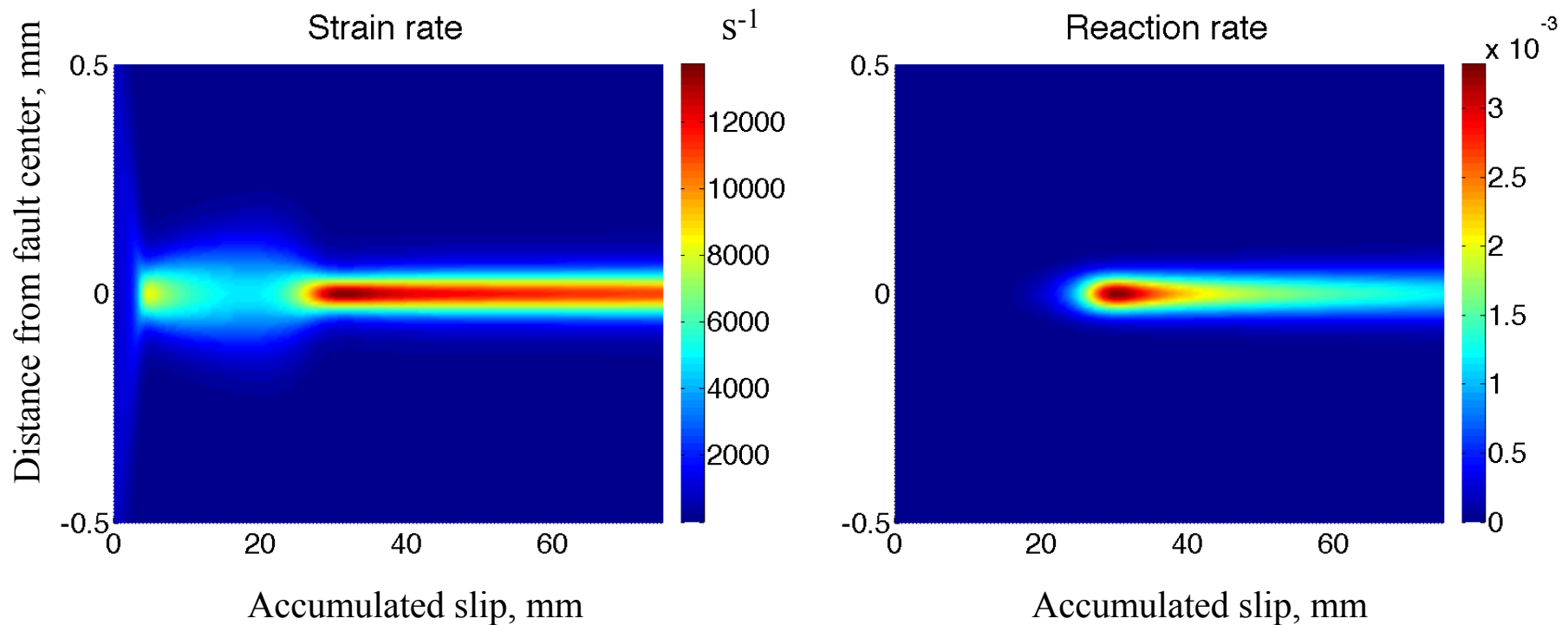
- When the reaction becomes important, we observe significant strain localization ($W_{\text{nonlin. calc.}}$ of order ~ 2 x $W_{\text{lin. pert.}}$).

Independence of reaction rate

• Our linear stability analysis predicts the localized zone width is independent of kinetic parameters. To test this we **increase *A* by three orders of magnitude.**
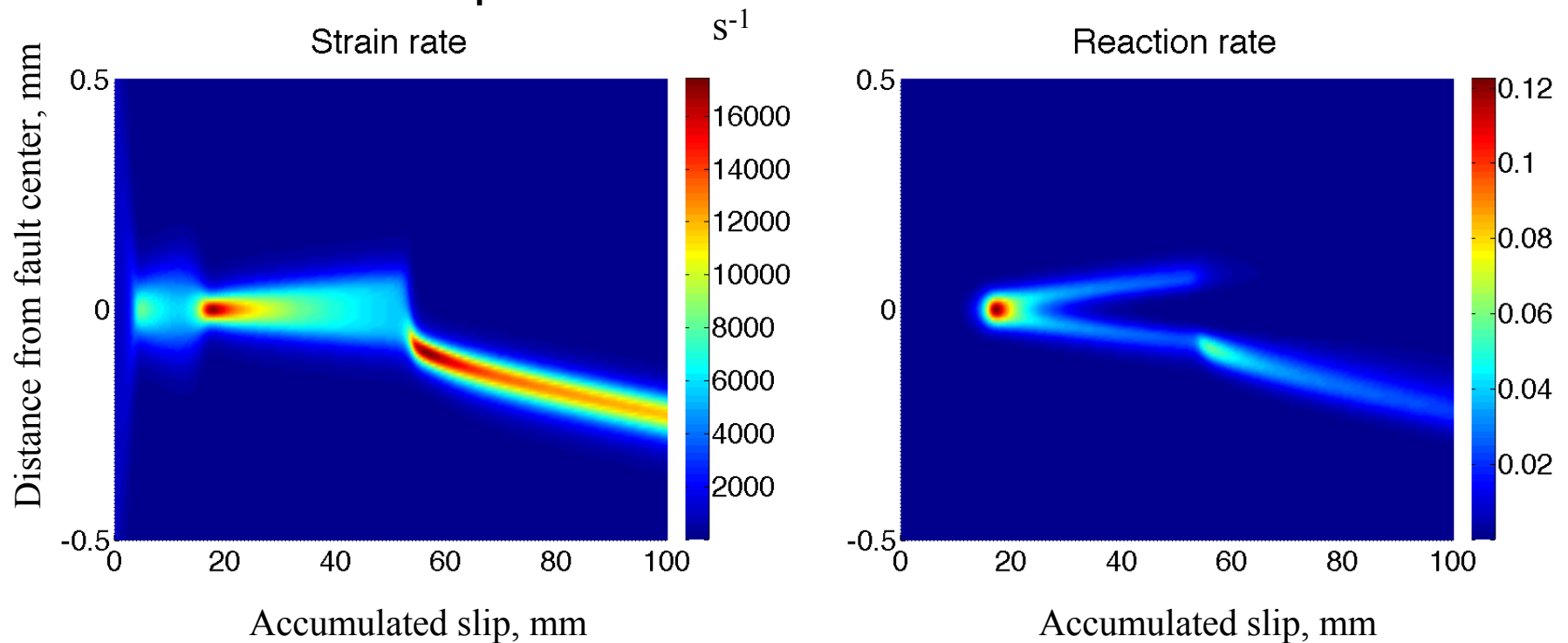

•The localized zone width has only decreased by a factor of two.

Platt, Brantut & Rice (AGU, Fall 2011; in prep 2013 for *JGR*)

Localized zone migration

- Now we investigate a case for which **depletion of reactant** is important.

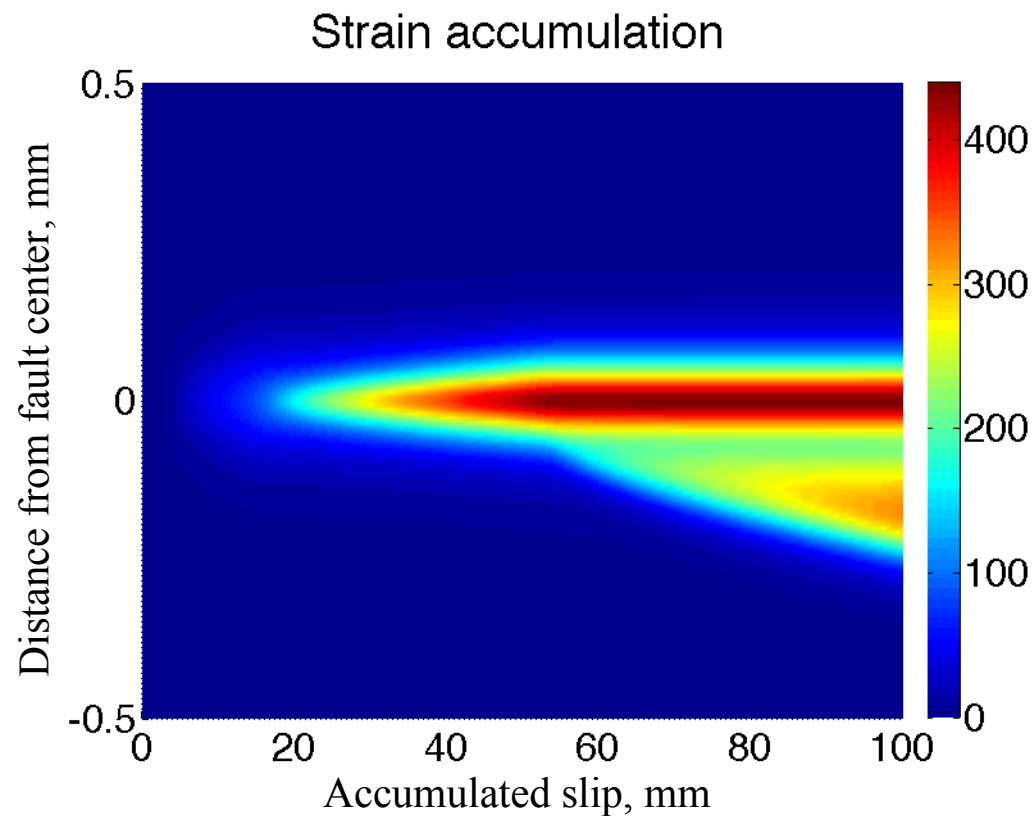Strain rate s^{-1}

Reaction rate

- Depletion causes the zone of localized straining to migrate. The strain rate and reaction rate profiles are strongly coupled.

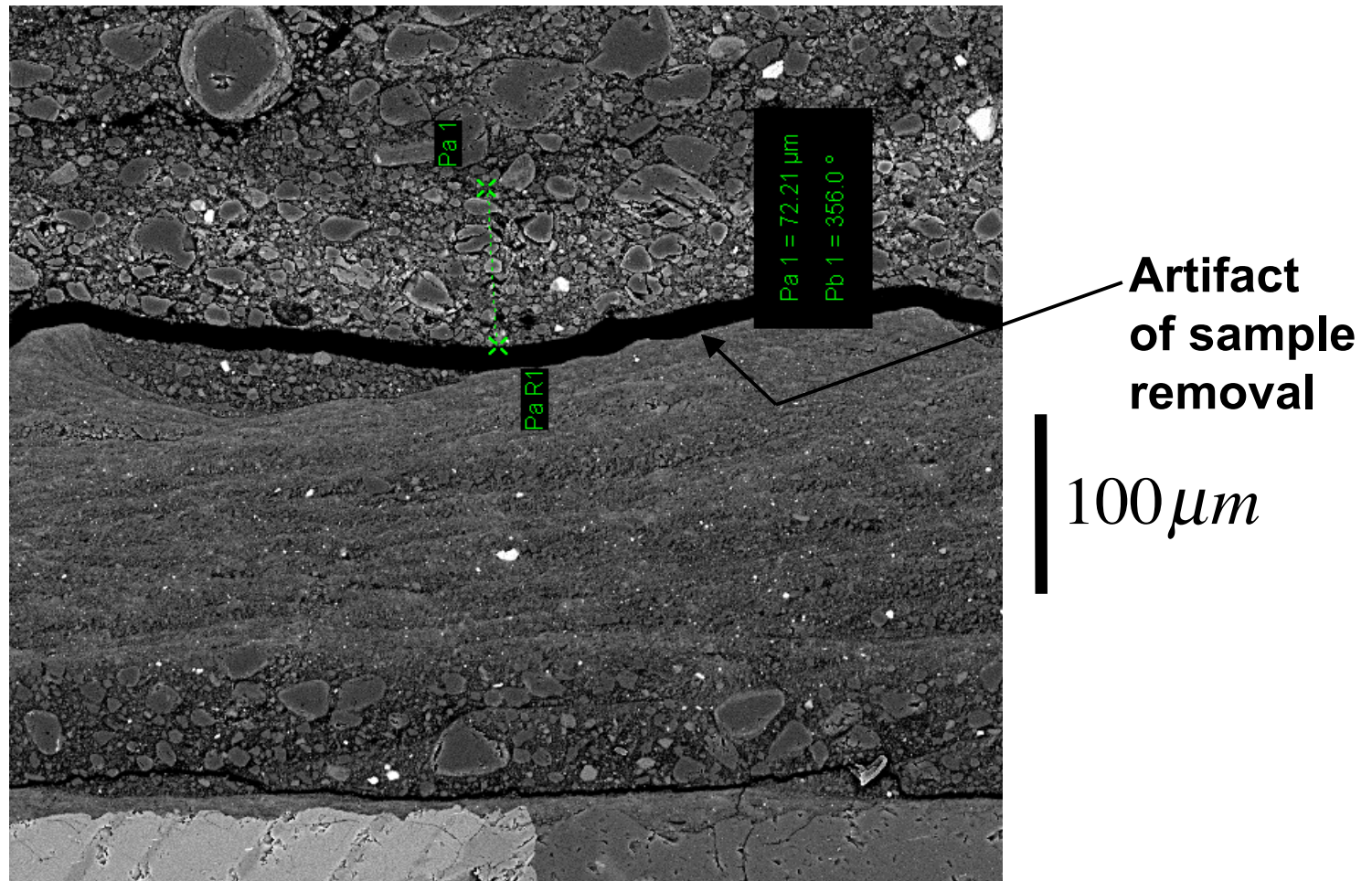Platt, Brantut & Rice (AGU, Fall 2011; in prep 2013 for *JGR*)

Localized zone migration

• This localized zone migration leads to a complex, non-monotonic, strain history.

Strain accumulation

Platt, Brantut & Rice (AGU, Fall 2011; in prep 2013 for *JGR*)

Observation suggesting migration
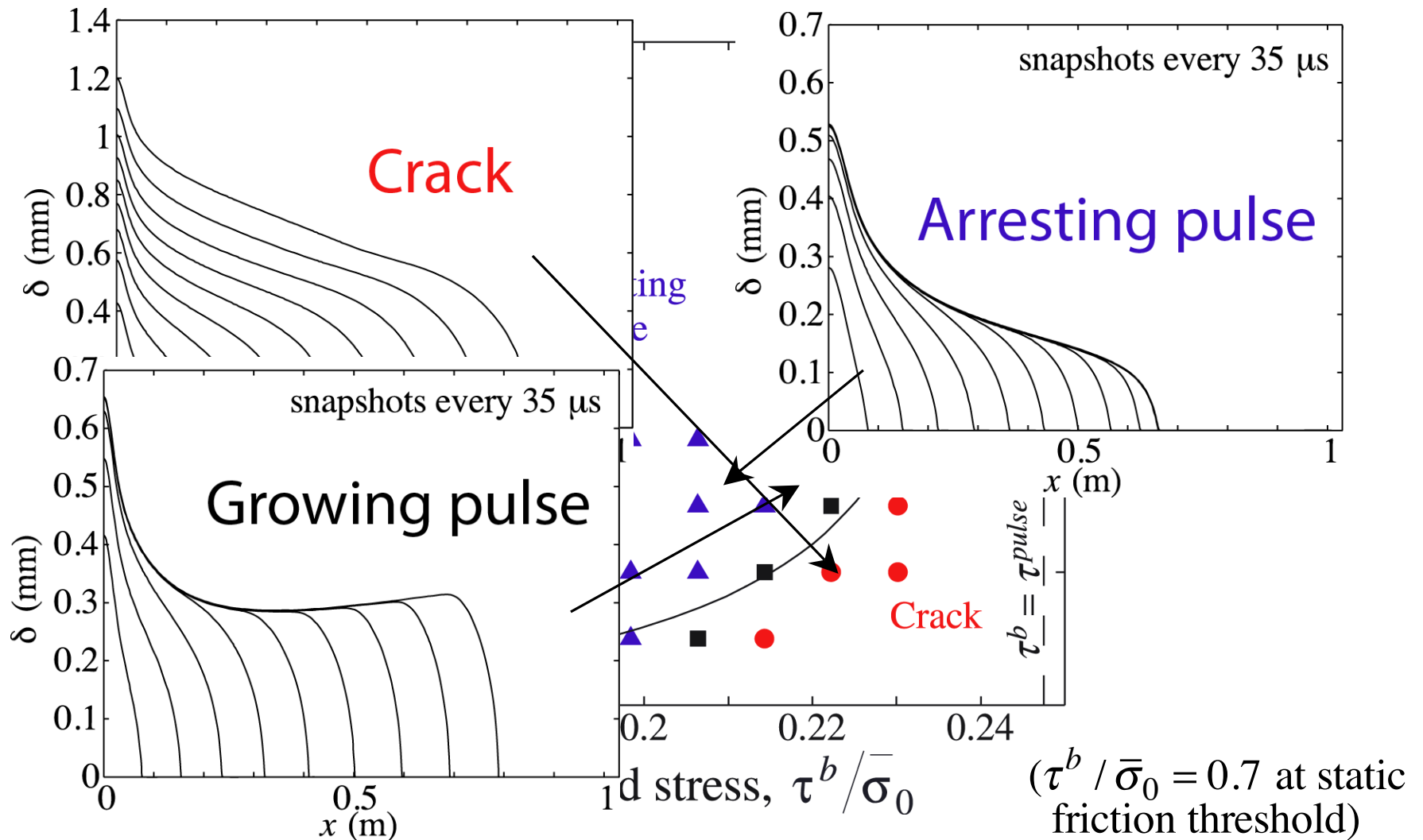# (in rotary shear of carbonate sample)

from T. Mitchell, Univ. Col. London (*private comm.*)

Noda, Dunham & Rice (JGR, 2009)

Effect of strong dynamic weakening (thermal pressurization + flash heating/weakening of frictional contacts) on when a rupture, once nucleated, can propagate to a large spatial extent.

$(\tau^b / \bar{\sigma}_0 = 0.7$ at static friction threshold)

Conclusions

- **Thermal Pressurization of Fluid (Water) Present In-Situ**
  - Linear stability analysis predicts very thin slip zones, width W ≈ 5 - 200 μm, within observed range.
  - Full nonlinear analysis also predicts very thin slip zones independent of initial layer thickness.
  - Localization causes additional weakening consistent with "slip on a plane" analysis
- **Thermal Pressurization by Decomposition Fluid**
  - Localization to comparable zones , W ≈ 2 - 180 μm width
  - Simulations indicated progress of reaction triggers localization
  - Width of localized zone is independent of kinetic parameters.
- **For a wide range of representative parameters, both processes lead to slip on very narrow zones (within the range of observations) causing additional dynamic weakening**

Concepts of fluid and solid mechanics, integrated with materials and thermal sciences, provide a valuable framework for addressing large-scale natural phenomena. We considered their applications to the following:

(1) **Ice sheet flow and subglacial hydrology:**
• Large iceberg calving as the enigmatic source of glacial earthquakes.
• Rapid glacial underflooding events as natural hydraulic fractures, like in a well-characterized spontaneous lake drainage on the Greenland Ice Sheet.
• Partial internal melting from shear heating as a control on flow resistance at the margins of rapidly flowing ice streams as on the Western Antarctic Ice Sheet.

(2) **Fault zones undergoing seismic slip:**
•Frictional heating leads to dynamic pressurization of pore fluids (native ground fluids, or decomposition products from clays and carbonates) within fault gouge.
•Result is dramatic shear-weakening results, with consequences for the dynamics of rupture propagation and earthquake phenomenology (strong but brittle faults, operating at low overall stress, no pronounced heat outflow, self-healing ruptures).

Common theme in cases considered:
Solid media considered are, or may become, fluid-infiltrated. Pressurization of the fluid weakens shear resistance and, in an extreme limit, may hydraulically fracture the host.