Shear heating and weakening of the margins of West Antarctic ice streams

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Ice streams are fast flowing bands of ice separated from stagnant ridges by shear margins. The mechanisms controlling the location of the margins remain unclear. We use published ice deformation data and a simple one-dimensional thermal model to show that West Antarctic ice stream margins have temperate ice over a substantial fraction of their thickness, a condition that may control their width. The model predicts a triple-valued relation between the thickness-averaged lateral shear stress and the lateral shear strain rate. Observed strain rates at the margins imply that they support slightly less lateral shear stress than adjacent ice within the stream. This requires enhanced basal resistance near the margin. We suggest, in agreement with the limited observations, the presence of a channelized drainage system at the margin that reduces the pore fluid pressure at the ice-till interface, thus increasing the shear stress acting on the yielding Coulomb-plastic bed.

1. Introduction

A complete collapse of the West Antarctic Ice Sheet would raise the global sea level by approximatively three to five meters [*Bamber et al.*, 2009; *Vaughan and Spouge*, 2002]. The West Antarctic ice streams (figure 1) are responsible for the lost of a large portion of this ice reservoir. They drain the ice into the Ross Ice Shelf at speeds of hundreds of meters per year, which is approximatively 2 to 3 orders of magnitude higher than the surrounding ice in the ridges [*Shabtaie and Bentley*, 1987]. While the width of ice streams may control the ice flow rate [*Van Der Veen and Whillans*, 1996], the mechanisms that select the location of the margins, and possibly allow them to migrate, are still uncertain. Since multi-decadal variations of their net mass flow rate, shown by *Joughin et al.* [2005], may have significant effects on global sea level, we investigate mechanisms that could control the location of ice stream margins.

The West Antarctic ice streams are about 1 km thick, active along hundreds of kilometers, are typically 30 to 80 km wide and have low surface slopes ($\sim 1.3\times10^{-3}$) [*Joughin et al.*, 2002]. The ice flow is driven by a relatively small gravitational driving stress ($\sim$ 10 kPa) that is equilibrated primarily through basal drag and laterally shear stressed zones at the margins [*Whillans and Van Der Veen*, 1993]. However, ice streams are underlain everywhere by a Coulomb-plastic bed with an extremely low yield strength ($\sim$ 1 to 5 kPa) due to a pore pressure in bed sediments that nearly equals the ice overburden pressure [*Tulaczyk et al.*, 2000; *Kamb*, 2001]. Hence, side drag provides most of the resistance to the driving stress [*Joughin et al.*, 2004]. At the margins of the ice streams, localized intense shear straining results in chaotic crevassing at the surface over few kilometers [*Harrison et al.*, 1998].

Due to surface crevasses, shear margins are difficult regions to access [*Harrison et al.*, 1998], and temperature measurements at depth are scarce. However, *Clarke et al.* [2000] conducted a radar survey at a abandoned shear margin in the Unicorn ridge and found diffractors in the ice sheet existing over 230 m above the bed, interpreting them as evidence for temperate ice. Additionally, theoretical models of margins show that the shear heating concentration at the boundary between temperate and frozen conditions at the bed induces onset of internal melting. That implies temperate ice extending up to a few hundred meters height and a few kilometers width [*Jacobson and Raymond*, 1998; *Schoof*, 2012; *Suckale et al.*, 2014]. *Suckale et al.* [2014] focused on Dragon margin of Whillans ice stream B2, showing that a close match to the downstream surface velocity profile [*Echelmeyer et al.*, 1994] could be achieved only in simulations which developed a large temperate zone within the margin. Here we investigate a large set of West Antarctic ice stream margins (map in figure 1), for which limited data is available [*Joughin et al.*, 2002] only as an averaged marginal strain rate over $\sim$ 2 km. Our results support the hypothesis that shear margins coincide with the development of temperate ice conditions.

If significant deformation-induced meltwater is generated at the margin, a basal channelized drainage operating a low water pressure [*Röthlisberger*, 1972; *Weertman*, 1972] may develop. Such channel lowers the pore fluid pressure at the base of the ice sheet in its vicinity. This increases the yield strength of the plastically deforming bed in the margin, which can limit ice stream widening as shown by *Schoof* [2004, 2012].

2. Model set up

Here we develop a model that leads us to infer temperate ice at the margins of nearly all West Antarctic ice streams. We apply our model to the margins of the sixteen transverse velocity profiles for ice streams reported by *Joughin et al.* [2002] and located in figure 1. Profiles beginning with the letter T are made at the tributaries of ice streams. For each profile, *Joughin et al.* [2002] provide the ice sheet thickness H, as extracted from a Digital Elevation Model [*Lythe and Vaughan*, 2001] and reported in table 1, as well as a thickness-averaged lateral shear stress. From the creep parameter they used, we invert for the shear strain rates $\dot{\gamma}_{lat}$ (transverse derivative of the downstream velocity) that were measured at the margins, as averaged over $\sim$ 2 km width (see table 1).

We consider an ice stream cross-section perpendicular to the direction of downstream flow. The ice stream is flowing in the downstream direction x, y is transverse to the flow and positive outward from the stream center, and z is

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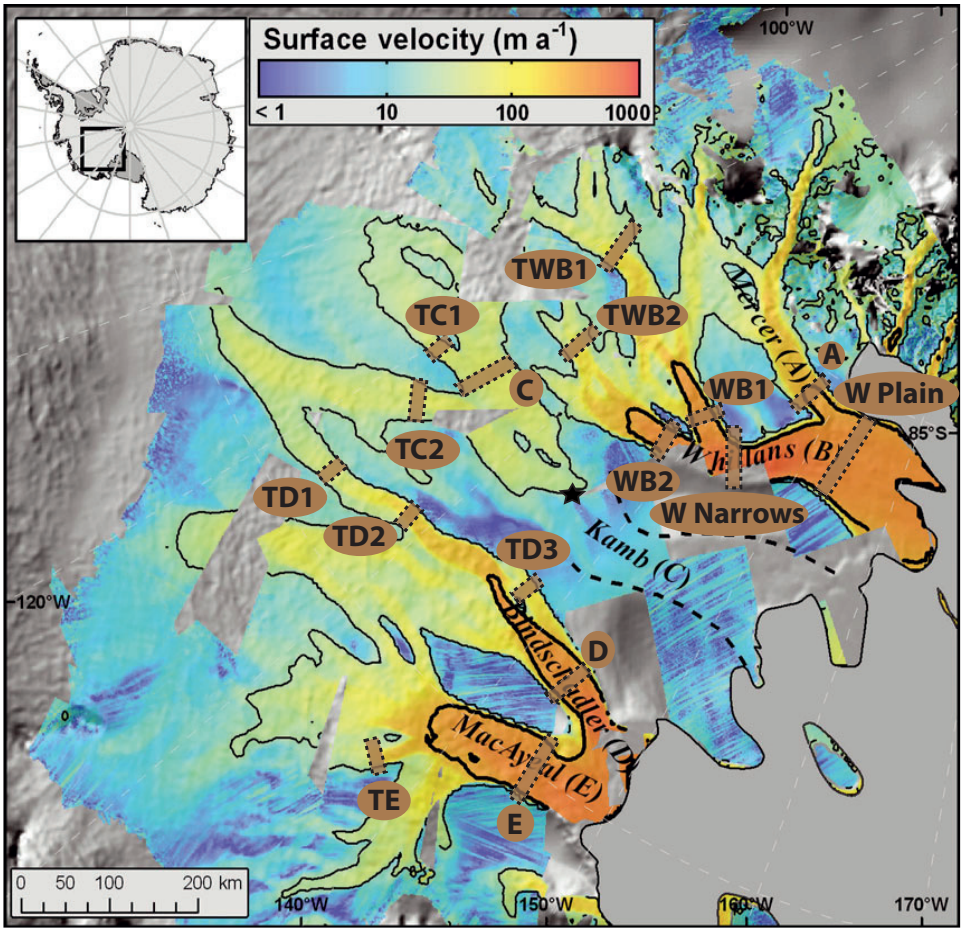

Figure 1. Surface velocity of West Antarctic ice streams, modified from *Le Brocq et al.* [2009]. Velocity contours shown are 25 m.yr^{-1} (thin line) and 250 m.yr^{-1} (thick line). *Joughin et al.* [2002] made profiles represented by brown lines and named in the brown ovals. The black star indicates the location of a borehole made at one shear margin of Kamb IS by *Vogel et al.* [2005].

the vertical coordinate, positive upward from the base of the ice (figure 2). The velocity components aligned with the (x, y, z) coordinates are (u, v, w). We define the lateral shear stress on vertical planes through the ice sheet oriented parallel to the flow as $\tau_{lat} = -\tau_{xy}$ and the basal shear stress as $\tau_{base} = \tau_{xz}$. This ice stream is driven by a gravitational driving stress $\tau_{grav} = \rho_{ice} g H \sin\alpha$ where ρ_{ice} is the ice density,

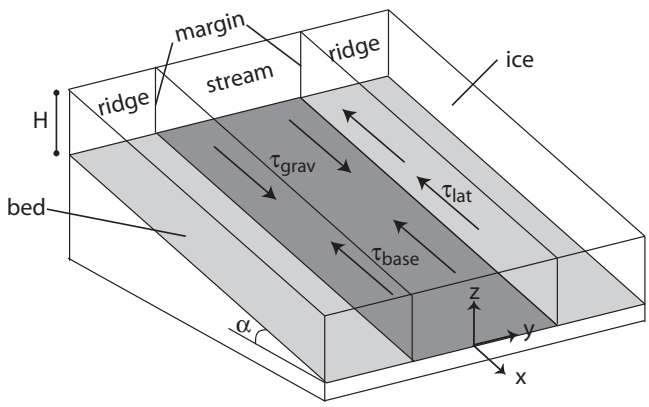

Figure 2. Sketch of the geometry assumed in this paper.

g the gravitational acceleration, H is the ice sheet thickness, and the surface slope $\sin\alpha$ measures the downward inclination of the slab. We consider a laterally constant thickness H and neglect any variation in net axial force in the sheet (see *Whillans and Van der Veen* [1997]). A force balance at a distance y from the center of an ice stream shows that the thickness-averaged $\bar{\tau}_{lat}$ increases with distance y from the center, $\bar{\tau}_{lat}(y)H = (\tau_{grav} - \bar{\tau}_{base})\, y$, where $\bar{\tau}_{base}$ is the basal resistive stress averaged over the width y. The lateral strain rate $\dot{\gamma}_{lat}$ transverse to the downslope direction, as predicted by Glen's law ($\dot{\gamma}_{lat} = 2A\bar{\tau}_{lat}^3$ where the "engineering" shear strain rate is $\dot{\gamma}_{lat} = -2\dot{\epsilon}_{xy}$), increases as $\bar{\tau}_{lat}^3$. Hence $\dot{\gamma}_{lat}$ scales as y^3 when we consider τ_{base} to be uniform. Then, the heating work rate $\bar{\tau}_{lat}\dot{\gamma}_{lat}$ associated with the lateral deformation increases as $\bar{\tau}_{lat}^4$, and hence, is roughly proportionally to y^4. With increasing y that becomes a significant heat source within the ice sheet and must ultimately induce internal melting (i.e., temperate ice) far from the center of the stream, if some process does not limit stream expansion.

The steady state temperature distribution T within an ice sheet is governed by

$$\rho_{ice} C_i \left(\vec{v} \cdot \nabla\right) T = \nabla \cdot \left(K \nabla T\right) + 2\tau_E \dot{\epsilon}_E, \qquad (1)$$

where the ice specific heat is C_i, the ice velocity vector is $\vec{v}$, the thermal conductivity is K, the effective shear stress τ_E is defined as the second invariant of the deviatoric stress

tensor $\tau_E^2 = \tau_{ij}\tau_{ij}/2$, the effective shear strain rate $\dot{\epsilon}_E$ is equal to the second invariant of the strain rate tensor $\dot{\epsilon}_E^2 = \dot{\epsilon}_{ij}\dot{\epsilon}_{ij}/2$. At the margin, the in-plane strain rate components are two orders of magnitude lower than the anti-plane strain rates [*Echelmeyer and Harrison*, 1999]. Therefore the shear heating $2\tau_E\dot{\epsilon}_E$ reduces to the work associated with anti-plane deformation of the ice $\tau_{lat}\dot{\gamma}_{lat}$. Rewriting in a one-dimensional (1D) approximation, $T = T(z)$, the equation governing the vertical temperature distribution in a column of ice at the margin is

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(K\frac{\mathrm{d}T}{\mathrm{d}z}\right) - \rho_{ice}C_i w\frac{\mathrm{d}T}{\mathrm{d}z} + \tau_{lat}\dot{\gamma}_{lat} = 0, \tag{2}$$

where w is the vertical (z direction) component of $\vec{v}$. The 1D approximation ignores lateral heat flux in the ice column. We discuss the role of transverse influx of colder ice from the ridge zone and margin migration (observed by *Harrison et al.* [1998] and *Echelmeyer and Harrison* [1999] at Dragon margin) in the discussion section.

We assume that the strain rate is uniform over depth [*Echelmeyer et al.*, 1994; *Scambos et al.*, 1994]. This is untenable where the ice sheet is frozen to the bed but a more acceptable assumption within the shear margin. Therefore the local volumetric rate of internal heat production in the ice column is

$$\tau_{lat}(\dot{\gamma}_{lat}, T)\,\dot{\gamma}_{lat} = 2A(T)^{-1/3}\left(\frac{\dot{\gamma}_{lat}}{2}\right)^{4/3}. \tag{3}$$

The temperature dependence of the creep parameter, thermal conductivity K and specific heat C_i are treated explicitly using the functions proposed by *Cuffey and Paterson* [2010] that follow results from field analyses and laboratory experiments. We adopt the vertical linear advection profile $w(z) = -a\,z/H$, where a is the accumulation rate at the top of the ice sheet. This was suggested by *Zotikov* [1986] and is commonly used in the glaciology literature (e.g., *Joughin et al.* [2002, 2004]; *Jacobson and Raymond* [1998]; *Suckale et al.* [2014]). A reasonable range in the region studied here is $a = 0.1$ to $0.2\,\mathrm{m.yr^{-1}}$ [*Giovinetto and Zwally*, 2000; *Spikes et al.*, 2004].

The governing equation of our 1D thermal model becomes

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(K(T)\frac{\mathrm{d}T}{\mathrm{d}z}\right) + \rho_{ice}C_i(T)\frac{az}{H}\frac{\mathrm{d}T}{\mathrm{d}z} + 2A(T)^{-1/3}\left(\frac{\dot{\gamma}_{lat}}{2}\right)^{4/3} = 0. \tag{4}$$

We assume a bed temperature at the pressure-dependent melting point T_{melt} following *Hooke* [2005], and a representative annual average atmospheric temperature $T_{atm} = -26$ °C at the surface [*Giovinetto and Zwally*, 2000].

Evaluating the creep parameter A and the thermal properties K and C_i at an 'average' column temperature $T_{avg} = (T_{melt} + T_{atm})/2$ we find an analytical solution for the vertical temperature profile (derivation in the supporting information),

$$\begin{aligned} T(z) &= T_{melt} + (T_{atm} - T_{melt})\,\frac{\mathrm{erf}\left[\sqrt{\mathrm{Pe}/2}\,(z/H)\right]}{\mathrm{erf}\left[\sqrt{\mathrm{Pe}/2}\right]} \\ &-\tau_{lat}\dot{\gamma}_{lat}H^2(K\,\mathrm{Pe})^{-1}\left[\int_0^1 \frac{\left[1-\exp\left(-\lambda\mathrm{Pe}\,z^2/2H^2\right)\right]}{\left[2\lambda\sqrt{1-\lambda}\right]}\,\mathrm{d}\lambda\right. \\ &\left.-\frac{\mathrm{erf}\left(\sqrt{\mathrm{Pe}/2}\,(z/H)\right)}{\mathrm{erf}\left(\sqrt{\mathrm{Pe}/2}\right)}\int_0^1 \frac{\left[1-\exp\left(-\lambda\,\mathrm{Pe}/2\right)\right]}{\left[2\lambda\sqrt{1-\lambda}\right]}\,\mathrm{d}\lambda\right], \end{aligned} \tag{5}$$

with the Péclet number $\mathrm{Pe} = aH/[K/(\rho_{ice}C_i)]$. This reduces to the solution already found by *Zotikov* [1986] when ignoring the shear heating rate $\tau_{lat}\dot{\gamma}_{lat}$. Unlike *Zotikov* [1986], equation (5) predicts temperatures in excess of melting at the base of ice stream margins due to the heat induced by shear (see supporting information).

Because the ice temperature cannot be greater than the melting point, we refine the thermal model capping the temperature at the melting point,

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}z}\left(K(T)\frac{\mathrm{d}T}{\mathrm{d}z}\right) &+ \rho_{ice}C_i(T)\frac{az}{H}\frac{\mathrm{d}T}{\mathrm{d}z} \\ &+ \left[1-\hat{H}(T-T_{melt})\right]2A(T)^{-1/3}\left(\frac{\dot{\gamma}_{lat}}{2}\right)^{4/3} \\ &= 0, \end{aligned} \tag{6}$$

with $\hat{H} \equiv$ Heaviside function, like in the formulation by *Suckale et al.* [2014]. Equation (6) assumes that all the shear heating generated in temperate ice is absorbed as latent heat to produce meltwater that is steadily evacuated. Mathematically, this problem takes the form of a free boundary problem in one dimension. The equation is solved subject to boundary conditions $T(z = H) = T_{atm}$ and $T(z = 0) = T_{melt}$. We denote by H' the thickness of temperate ice. We solve this model numerically using Runge-Kutta methods and shooting techniques to calculate the fraction of the ice sheet that is temperate, H'/H.

3. Temperate ice at the margins of West Antarctic ice streams

We find that six margins of the seven active ice streams (excluding the inactive Kamb) are at a level of strain rate at which internal melting occurs (see ratio H'/H in table 1). The temperature profiles predicted by equation (6) are shown in figure 3. The strain rate level measured at Bindschadler ice stream ($0.058\,\mathrm{yr^{-1}}$ at the margins of profile D) is not enough to melt a portion of the 888 m ice thickness, although our model predicts onset of melting (i.e.,

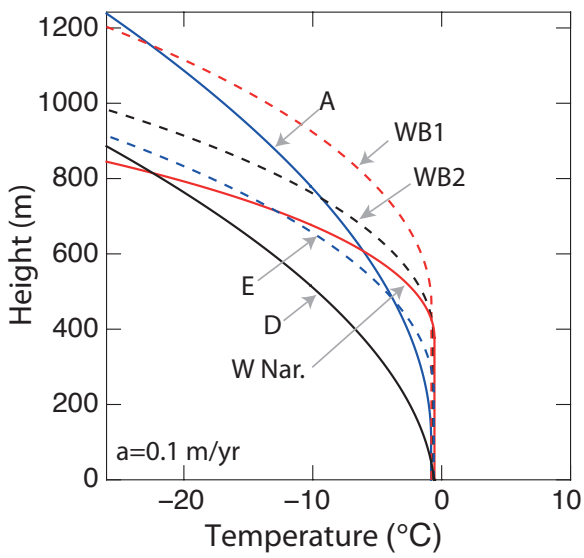

Figure 3. Temperature profiles at the margins of the ice stream profiles located in figure 1 and found by solving numerically equation (6) with temperature-dependent ice properties. Most of them imply a substantial thickness of temperate ice adjoining the bed.

Table 1. Ice sheet thickness H and shear strain rates $\dot{\gamma}_{lat}$ at the margins of the profiles located in figure 1 [*Joughin et al.*, 2002]. Temperate ice height fractions H'/H are predicted by our 1D thermal model, equation (6), for $a = 0, 0.1$, and $0.2\,\mathrm{m.yr^{-1}}$.

Ice Stream	Profile	H (m)	$\dot{\gamma}_{lat}$ (10^{-2}.yr^{-1})	H'/H (%) $a = 0\,\mathrm{m.yr^{-1}}$	H'/H (%) $a = 0.1\,\mathrm{m.yr^{-1}}$	H'/H (%) $a = 0.2\,\mathrm{m.yr^{-1}}$
Mercer	A	1242	4.2	23	9	0
Whillans	WB1	1205	7.0	43	39	32
	WB2	985	9.5	43	39	34
	W Narrows	846	13.5	47	45	42
	W Plain	735	5.1	0	0	0
	TWB1	2188	3.8	54	50	43
	TWB2	1538	4.0	36	26	10
Kamb	C	1805	1.0	0	0	0
	TC1	1802	1.4	0	0	0
	TC2	2196	0.9	0	0	0
Bindschadler	D	888	5.8	11	0	0
	TD1	1952	2.5	32	16	0
	TD2	1412	5.4	43	37	29
	TD3	1126	2.2	0	0	0
MacAyeal	E	916	8.1	32	26	16
	TE	1177	5.5	32	23	10

$H'/H = 0^+$) for a 5% increase in strain rate to 0.061 yr^{-1}. In fact the upper confidence limit of strain rate measured by *Joughin et al.* [2002] corresponds to $H'/H = 7\,\%$. Also, the maximum strain rate observed at the southern shear zone increases by $\sim 280\,\%$ to $0.16\,\mathrm{yr^{-1}}$ only 30 km downstream [*Scambos et al.*, 1994]. For a comparable ice thickness to what has been measured at profile D, that level of strain rate would melt 53 % of the ice sheet thickness according to our model. The margins of profile W Plain, located further downstream than any other profile on Whillans ice stream, are not predicted to be temperate. However, at this location, the ice stream is extremely flat and wide, possibly making its characteristics different from other West Antarctic ice streams [*Bindschadler et al.*, 2005].

We find that five margins of the six active ice stream tributaries (excluding the dormant Kamb) reach the melting point so that $H'/H \geq 0$ (table 1). The model does not predict temperate ice at the margins of Kamb ice stream tributaries. This is not surprising since the ice flow does not have the characteristics of streaming flow (e.g., it has a low marginal strain rate, approximately 0.01 yr^{-1}).

We conducted a sensitivity analysis of the model. We display H'/H in table 1 in the case of zero vertical advection – corresponding to when the downslope stretching matches the transverse compression – and for a doubled accumulation rate, $a = 0.2$ m.yr^{-1}. Predictions of temperate ice at the margins are found for the range of accumulation rates considered.

4. Triple-valued lateral shear stress law

Now we examine the lateral shear stress carried by a margin that is temperate and describe a hypothesis for the role of marginal temperate ice on ice stream dynamics. The average lateral shear stress over the ice sheet thickness $\bar{\tau}_{lat}$, neglecting any porosity effect on strength in the temperate region, is

$$\bar{\tau}_{lat} = \frac{1}{H}\int_0^H A(T(z))^{-1/3}\mathrm{d}z\,\left(\frac{\dot{\gamma}_{lat}}{2}\right)^{1/3}. \qquad (7)$$

Our thermal model sets the temperature profile $T(z)$. For a given H, equation (7) implies a triple-valued relationship between $\bar{\tau}_{lat}$ and $\dot{\gamma}_{lat}$. In a certain range of shear stress, three values of shear strain rate correspond to the same value of shear stress (figure 4). The shear stress increases with increasing strain rate before onset of melting. Once the work associated with lateral deformation of ice is sufficient to melt the ice at the base ($H'/H > 0$), $\bar{\tau}_{lat}$ decreases due to thermal softening. For larger strain rates, strain rate hardening becomes dominant and $\bar{\tau}_{lat}$ then increases with $\dot{\gamma}_{lat}$. We find such a triple-valued law for $H > 300\,\mathrm{m}$ with an accumulation rate of $0.1\,\mathrm{m.yr^{-1}}$ and $H > 200\,\mathrm{m}$ with $a = 0.2\,\mathrm{m.yr^{-1}}$. Since $H > 800\,\mathrm{m}$ for all West Antarctic ice streams, we expect a triple-valued law in general. We perform simulations using the thicknesses listed in table 1 and display nine of the sixteen profiles such that the thicknesses used for the simulations span the entire range of H (figure 4). Most of the West Antarctic ice stream margins experience a level of straining at which we predict the side drag to sit in the region of locally reduced shear stress due to thermal softening. When entering the margin the column of ice supports lower stresses than at the peak stress associated with onset of a temperate zone.

A force balance of an ice stream that ignores transverse variation of ice thickness and variation in net axial force in the sheet [*Whillans and Van Der Veen*, 1993] gives

$$\tau_{grav} - \tau_{base} = H\frac{\mathrm{d}\bar{\tau}_{lat}}{\mathrm{d}y} = H\frac{\mathrm{d}\bar{\tau}_{lat}}{\mathrm{d}\dot{\gamma}_{lat}}\frac{\mathrm{d}\dot{\gamma}_{lat}}{\mathrm{d}y}. \qquad (8)$$

The gravitational driving stress is assumed uniform. Data for profiles D [*Scambos et al.*, 1994] and WB2 [*Echelmeyer et al.*, 1994] show that $\mathrm{d}\dot{\gamma}_{lat}/\mathrm{d}y$ is strongly positive over 4 km bands near the margin whereas $\mathrm{d}\bar{\tau}_{lat}/\mathrm{d}\dot{\gamma}_{lat} < 0$ (figure 4) over some of that $\dot{\gamma}_{lat}$ range. That implies a local basal strengthening (with $\tau_{base} > \tau_{grav}$ locally). For a locally uniform τ_{base}, integration of equation (8) over y shows that a lateral shear stress drop of $\Delta\bar{\tau}_{lat} = -2$ to $-5\,\mathrm{kPa}$ over a distance $\delta \approx H$ in the margin corresponds to a high basal shear stress $\tau_{base} = \tau_{grav} - \Delta\bar{\tau}_{lat}\,H/\delta \approx 12$ to $15\,\mathrm{kPa}$, assuming a typical gravitational driving stress of $10\,\mathrm{kPa}$ for West Antarctic ice streams [*Joughin et al.*, 2002].

We hypothesize that bed strengthening in the margin is related to the development of a channelized drainage operating at low water pressure [*Röthlisberger*, 1972; *Weertman*, 1972]. Such is supported by the observation of a 1.6 m water-filled cavity [*Vogel et al.*, 2005] between the bottom of the ice sheet and the bed at the margin of Kamb ice stream

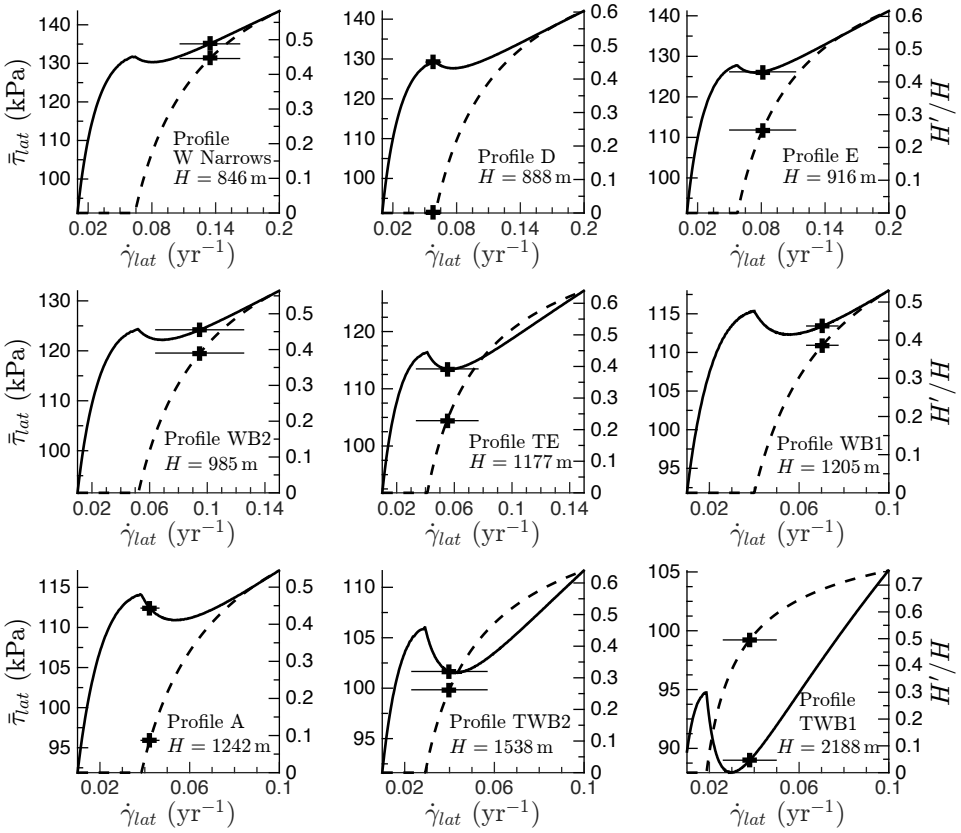

Figure 4. Triple-valued lateral shear stress (solid line) and temperate fraction of the ice sheet thickness (dashed line) versus shear strain rate. Simulations are performed for the profiles listed in table 1 using their corresponding thicknesses (ordered by increasing H here). We also indicate the shear strain rate measured by *Joughin et al.* [2002] at the margins (black crosses with error bars).

(borehole location indicated by a star in figure 1), and is a plausible consequence of liquid production in the deforming temperate regions predicted here. The reduced pore pressure, due to the presence of a channelized drainage, increases the shear strength τ_{base} of the till in its vicinity and provides a local strengthening of the bed consistent with the enhanced τ_{base} implied by equation (8).

5. Discussion

Surface crevassing at ice stream margins provides a further link between deformation and internal heating. *Harrison et al.* [1998] used thermistors frozen into boreholes to measure ice temperature in the upper half of the ice sheet at ten locations in the vicinity of the southern margin of Dragon margin. They showed that the cold winter air pooling in the crevasses makes the margin $\sim 10^{\circ}$C colder than the adjacent uncrevassed ridge and stream over the first thirty meters depth. The temperature-depth profiles show that at intermediate depths the ice temperature is no longer influenced by surface crevassing, and below a depth of 150 m the ice temperature within the margin is higher than the temperature in the ridge.

Cold air cools the top of the ice sheet and hence, strengthens the ice. However, a heavily crevassed margin supports lower lateral stresses at the surface. A cracked material is mechanically softer and forces the lower portion of the margin to support larger stresses. Shear heating focuses in the lower portion of the margin and raises the temperature profile predicted by our thermal model. Future work will be needed to address the effect of crevasses on the margin temperatures.

While our model operates at steady state, which requires time for the ice to warm due to shear heating, there are inferences of migration of Dragon margin [*Echelmeyer and Harrison*, 1999] based on repeated measurements of the surface velocity profile. Although their study does not consider the effect of the gradual opening and rotation of the crevasses on the time evolution of the velocity profile, *Echelmeyer and Harrison* [1999] estimated a migration rate of approximately ten meters per year. In addition, they showed observations of lateral inflow of ice from both the stream and the ridge into the margin. A simplified scaling analysis gives an ice resident time at the margin of approximately $t_{res} = \rho_{ice} C_i H^2/K = 10^4$ yr while *Harrison et al.* [1998] argued more likely for half a century. Both the steady state assumption and the lack of lateral heat flux are restrictions of our 1D thermal model that may overestimate the thickness of temperate ice. Our model, however, has reasonable agreement with the 2D model developed by *Suckale et al.* [2014] (see their figure 7 for a comparison) that includes lateral heat flow. However, if the migration is at first outward and then inward the resident time of the ice inside the shear margin would become more important and our estimated temperature profiles more realistic. This is consistent with the observation of a complex deformation history that occurred over the last few hundred years at Dragon margin [*Clarke et al.*, 2000]. Furthermore, *Suckale et al.* [2014] found it impossible to closely match the ice deformation data when incorporating significant lateral advection of ice in their model.

6. Conclusion

We present simple thermo-mechanical models of the West Antarctic ice streams that suggest that their margins coincide with the development of temperate ice conditions within the ice sheet. We show that when the ice deformation-heating work at the margins is incorporated in a 1D vertical heat transfer analysis (as in *Zotikov* [1986]), the result typically predicts temperatures in excess of the melting temperature. We, thus, produce a still 1D, but more refined thermal model of the margins, with a full temperature dependence of ice properties and allowing for a temperate zone adjoining the bed. Using published ice sheet deformation and thickness data [*Joughin et al.*, 2002], this model predicts that most margins of the active West Antarctic ice streams are in a state of partial melt, with temperate ice being present over a fraction of the ice height.

Our thermal model of the margins implies a triple-valued relationship between the average lateral shear stress supported by a column of ice and the lateral shear strain rate. A temperate margin supports less lateral shear stress than the adjacent, not yet temperate, ice of the more rapidly moving stream and, hence, the basal resistance is enhanced near the margin. We suggest that the subglacial drainage system at the margin increases the basal shear strength and may be a primary factor limiting lateral expansion of the stream.

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Supporting Information for "Shear heating and weakening of the margins of West Antarctic ice streams"

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Contents of this file

1. Analytical solution of the 1D thermal model for temperature-independent ice properties

2. Temperature profiles at the margins of West Antarctic ice streams predicted by the analytical solution of the 1D thermal model for temperature-independent ice properties

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Introduction This Supporting Information includes the mathematical derivation of the analytical solution to the 1D thermal model considering temperature-independent ice properties and the temperature profiles of the West Antarctic ice stream margins predicted by this solution.

Analytical solution of 1D thermal model for temperature-independent ice properties

To solve

$$K\frac{\mathrm{d}^2T}{\mathrm{d}z^2} + \rho_{ice}C_i\frac{az}{H}\frac{\mathrm{d}T}{\mathrm{d}z} + \tau_{lat}\dot{\gamma}_{lat} = 0, \tag{1}$$

we define $G = (\mathrm{d}T/\mathrm{d}z)$, $\eta = (z/H)^2$, $S = \tau_{lat}\dot{\gamma}_{lat}$ and $\mathrm{Pe} = \rho_{ice}C_iaH/K$, the Péclet number. Then, the previous equation reduces to,

$$2K\frac{\mathrm{d}G}{\mathrm{d}\eta} + \rho_{ice}C_iaHG = -\frac{SH}{\sqrt{\eta}}, \tag{2}$$

and integration gives,

$$\begin{aligned} G &= G_0\exp(-\mathrm{Pe}\,z^2/2H^2) \\ &\quad -\frac{S}{K}\int_0^z \exp\left[-\mathrm{Pe}(z^2-\tilde{z}^2)/2H^2\right]\mathrm{d}\tilde{z}, \end{aligned} \tag{3}$$

with $G_0 = (\mathrm{d}T/\mathrm{d}z)_{z=0}$. We integrate the latter relation using $T(z=0) = T_{melt}$ and find

$$\begin{aligned} T(z) &= T_{melt} + \sqrt{\frac{\pi}{2\mathrm{Pe}}}HG_o\,\mathrm{erf}\left(\frac{\sqrt{\mathrm{Pe}/2}\;z}{H}\right) \\ &\quad -\frac{S}{K}\int_0^z\int_0^{\hat{z}} \exp\left[\mathrm{Pe}(\tilde{z}^2-\hat{z}^2)/2H^2\right]\mathrm{d}\tilde{z}\mathrm{d}\hat{z}. \end{aligned} \tag{4}$$

Using polar coordinates in the integrations in the $\hat{z}, \tilde{z}$ plane, the analytical temperature distribution along the z-axis is

$$\begin{aligned} T(z) &= T_{melt} + \sqrt{\frac{\pi}{2\mathrm{Pe}}}HG_o\,\mathrm{erf}\left(\frac{\sqrt{\mathrm{Pe}/2}\;z}{H}\right) \\ &\quad -\frac{SH^2}{K\,\mathrm{Pe}}\int_0^{\pi/4}\frac{1-\exp\left(-\frac{\mathrm{Pe}\cos(2\theta)}{2\cos^2(\theta)}\frac{z^2}{H^2}\right)}{\cos(2\theta)}\mathrm{d}\theta. \end{aligned} \tag{5}$$

Rewriting the latter integral, the temperature profile along the z-axis is

$$T(z) = T_{melt} + \sqrt{\frac{\pi}{2\mathrm{Pe}}}HG_0\,\mathrm{erf}\left(\sqrt{\mathrm{Pe}/2}\;\frac{z}{H}\right) - \frac{SH^2}{K\,\mathrm{Pe}}\int_0^1\frac{1-\exp\left(-\lambda\mathrm{Pe}z^2/2H^2\right)}{2\lambda\sqrt{1-\lambda}}\mathrm{d}\lambda \tag{6}$$

and G_0 is found matching the second boundary condition $T(z = H) = T_{atm}$,

$$G_0 = \frac{2\sqrt{\mathrm{Pe}/2}}{\sqrt{\pi} H \,\mathrm{erf}\left(\sqrt{\mathrm{Pe}/2}\right)} \Bigg(T_{atm} - T_{melt} + \frac{S H^2}{K\,\mathrm{Pe}} \int_0^1 \frac{1 - \exp\left(-\lambda \mathrm{Pe}/2\right)}{2\lambda\sqrt{1-\lambda}}\, \mathrm{d}\lambda \Bigg) \tag{7}$$

This gives the solution written in equation (5) of the main paper

$$\begin{aligned} T(z) &= T_{melt} + (T_{atm} - T_{melt})\, \mathrm{erf}\left[\sqrt{\mathrm{Pe}/2}\,(z/H)\right] / \,\mathrm{erf}\left[\sqrt{\mathrm{Pe}/2}\right] \\ &- \tau_{lat}\dot{\gamma}_{lat} H^2 (K\,\mathrm{Pe})^{-1} \Bigg[\int_0^1 \left[1 - \exp\left(-\lambda \mathrm{Pe}\, z^2/2H^2\right)\right] / \left[2\lambda\sqrt{1-\lambda}\right] \mathrm{d}\lambda \\ &- \mathrm{erf}\left(\sqrt{\mathrm{Pe}/2}\,(z/H)\right) / \,\mathrm{erf}\left(\sqrt{\mathrm{Pe}/2}\right) \int_0^1 \left[1 - \exp\left(-\lambda\,\mathrm{Pe}/2\right)\right] / \left[2\lambda\sqrt{1-\lambda}\right] \mathrm{d}\lambda \Bigg]. \end{aligned} \tag{8}$$

Temperature profiles at the margins of West Antarctic ice streams predicted by the analytical solution for temperature-independent ice properties

We consider a set of six active ice stream profiles (A, WB1, WB2, W Narrows, D, E). Using a value of 0.1 m.yr^1 for the accumulation rate [*Giovinetto and Zwally*, 2000; *Spikes et al.*, 2004], and the ice sheet thickness H and shear strain rate $\dot{\gamma}_{lat}$ from table 1 of the main paper we find that the lower part of the temperature profiles at the margins predicted by equation (8) have temperatures in excess of the melting point (see figure 1).

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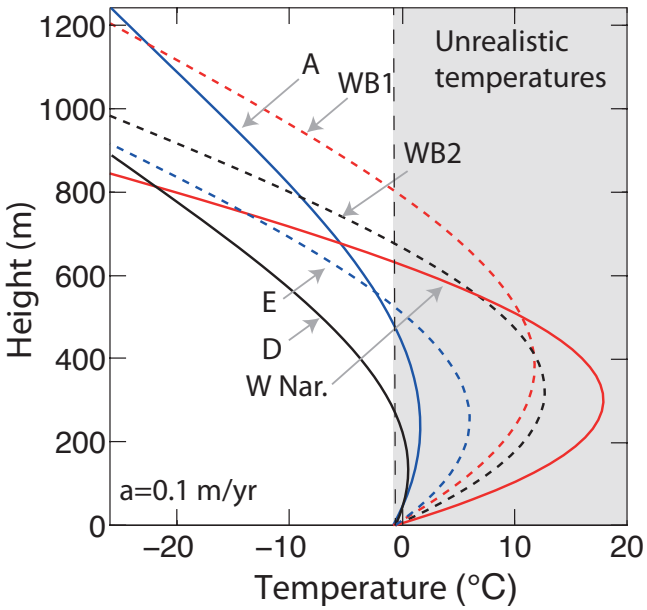

Figure 1. Analytically predicted (equation (8)) temperature profiles for temperature-independent ice properties at the margins of ice streams, showing unacceptable $T > T_{melt}$. The parameters for each profile are in table 1 of the main manuscript.