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SOME STUDIES OF CRACK DYNAMICS

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Abstract

Recent developments in fracture dynamics include the discovery of elastic waves which propagate along moving crack fronts in three-dimensional solids, and the identification of possible sources of the roughening and low terminal speeds of tensile cracks in brittle amorphous solids. Those topics are discussed briefly here.

Keywords: fracture / crack dynamics / elastodynamics

1. Introduction

The focus in this brief report is on two areas of tensile crack dynamics in elastic-brittle solids that have received recent focus. These involve the crack front waves revealed in numerical and analytical studies of cracking along a plane in a 3D solid, and the problem of understanding the significantly sub-Rayleigh limiting speeds of tensile cracks in brittle amorphous solids, with associated clusters of microcracks and roughening of the fracture surface. A related discussion of recent studies on the dynamics of crack and fault rupture is given in [1]. That includes some additional topics that were covered in the oral version of this presentation, on dynamic frictional slip, especially the unstable form it takes along interfaces between elastically dissimilar materials, on slip-rupture along earthquake faults, and on combined tensile and shear failures at bimaterial interfaces.

Here tensile cracking is addressed in the framework of continuum elastodynamics. The governing equations are the equations of motion $\nabla \cdot \boldsymbol{\sigma} = \rho \partial^2 \mathbf{u} / \partial t^2$ and the stress - displacement gradient relations $\boldsymbol{\sigma} = \lambda(\nabla \cdot \mathbf{u})\mathbf{I} + \mu[(\nabla \mathbf{u}) + (\nabla \mathbf{u})^T]$. Two different fracture formulations are used. The first is a singular crack model, in which one sets $\sigma_{yy} = 0$ on the mathematical cut along $y = 0$ (see lower part of Figure 1) which is the crack surface. That leads to a well known singular field of structure

$$\lim_{r \to 0} \left(\sqrt{r}\sigma_{\alpha\beta}\right) = \sqrt{\mu G}\,\Sigma_{\alpha\beta}(\theta, v / c_s, c_p / c_s)$$

where r, θ are polar coordinates at the crack tip, v is the speed of crack propagation, and the $\Sigma_{\alpha\beta}$ are dimensionless universal functions [2,3]. Here c_p, c_s are the body wave speeds. The strength of the singularity has been normalized in terms of G, which is the energy release rate (energy flow to crack tip singularity, per unit of new crack area), expressed by

$$G = \lim_{\Gamma \to 0} \int_{\Gamma} \left[n_x (W + \tfrac{1}{2} \rho \, | \partial \mathbf{u} / \partial t |^2) - \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \partial \mathbf{u} / \partial x \right] ds \ .$$

In that expression, W is the strain energy density, coordinate x points in the direction of crack growth, Γ is a circuit which loops around the crack tip at the place of interest, and s is arc length along Γ, whose outer normal is $\mathbf{n}$.

From the Freund [4] solution for the unsteady tensile crack motion it is known that G has the structure $G = g(v(t))G_{rest}$. Here $g(v)$ is a universal function of crack speed v. It satisfies $g(0^+) = 1$ and diminishes monotonically to $g(c_R) = 0$ at the Rayleigh speed c_R, which is therefore the theoretical limiting speed at least so long as the crack remains on a plane (see below). The term represented by G_{rest} is a complicated and generally untractable functional of the prior history of crack growth and of external loading, but is independent of the instantaneous crack speed $v(t)$. Owing to that structure for G, it is possible for cracks to instantaneously change v if the requisite energy supply, G_{crit}, for cracking changes discontinuously along the fracture path. For a solid loaded by a remotely applied tension, it will generally be the case that G_{rest} increases as the crack lengthens. It also increases, quadratically, with the intensity of the applied stress. Thus, if G_{crit} is bounded and if the energy flows into a single crack tip (e.g., single crack moving along a plane), then in a sufficiently large body, $G_{crit} / G_{rest} \to 0$ as the crack lengthens. Since that ratio is just $g(v)$, so also will $g(v) \to 0$, which means that v will accelerate towards the value c_R at which $g(v) = 0$. That is the sense in which c_R is the limiting crack speed. In the simplest model, which includes the classical Griffith model, we take G_{crit} as a material constant, although more realistically G_{crit} must be considered as a function of crack speed v, to be determined empirically or by suitable microscale modeling.

An alternative to the singular model is the Barenblatt-Dugdale cohesive zone fracture formulation, a displacement-weakening model with finite stresses. It is often more congenial for numerical simulation, even in cases for which the process zone is small enough that one would be happy to use the singular crack model. It provides for gradual decohesion by imposing a weakening relation between the tensile stress σ_{yy} and displacement-discontinuity δ_y on the crack plane $y = 0$. See the lower portion of Figure 1. The singularity at the crack tip is then spread into a displacement weakening zone. Its length R (measured in the direction of crack growth) scales [5,6] as, roughly, $\mu\delta_o / [\sigma_o f(v)]$ with σ_o being the maximum cohesive strength and δ_o the displacement at which cohesion is lost, and with $f(v)$ being a universal function [5,6] satisfying $f(0^+) = 1$ and $f(c_R) = \infty$. (The latter limit poses a challenge for numerical simulation of fracture at speeds very close to c_R.) When R << all length scales in the problem (crack length, distance of wave travel, etc.), predictions of the displacement-weakening model agree with those of the singular crack model, with G_{crit} identified (Figure 1) as the area under the cohesive relation.

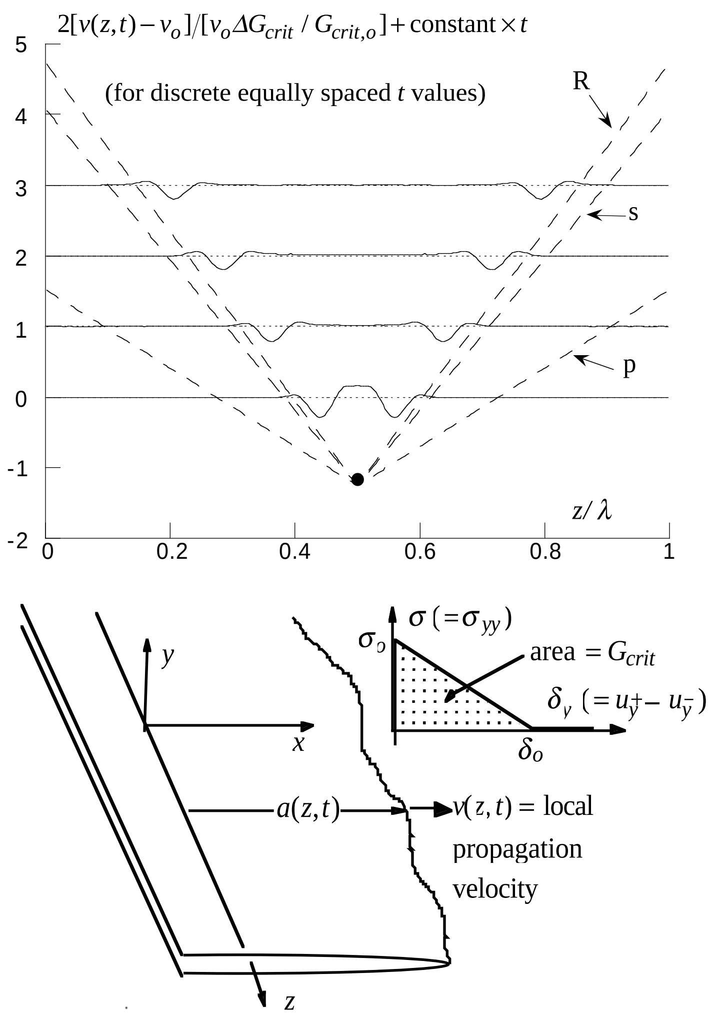

Figure 1. Cracking along a plane in a 3D elastic solid, and (at top) results of numerical simulation [6] showing crack front wave.

2. Crack Front Waves

Figure 1 shows, in its lower part, a tensile crack growing along the plane $y = 0$ in a 3D solid. Earlier work by the author and coworkers [7,8] addressed a simplified version of this problem for a model elastic theory. That involved a single displacement component u satisfying a scalar wave equation, in a 3D solid with a planar crack of non-straight front. It shows $1/\sqrt{r}$ crack tip singularities in ∇u, of a speed-dependent angular structure somewhat similar to those of actual elasticity. An energy release rate G, as well as a cohesive zone formulation, can be defined for that scalar model. Its solution revealed that there was a long-lived response to a local perturbation of the crack front. Such perturbation could be generated by having the crack pass through a region where the fracture energy was modestly different than elsewhere, although in the case of the scalar model the crack ultimately recovers a perfectly straight front.

That work created interest in addressing such problems in the context of actual elasticity. Willis and Movchan [9] soon produced the corresponding singular crack solution, for a crack whose front position $x = a(z,t)$ is linearly perturbed from $x = v_o t$, that is, from a straight front moving at uniform speed v_o. The fuller implications of their solution were revealed only later by Ramanathan and Fisher [10], confirming what had been suggested from spectral numerical simulations of Morrissey and Rice [11,6] in the framework of the cohesive zone model: For crack growth in a perfectly elastic solid with a constant fracture energy G_{crit}, perturbation of the crack front leads to a wave which propagates laterally, without attenuation or dispersion, along the moving crack front. That wave has been called a crack front wave.

Figure 1 shows in its upper part the numerical results [6] suggesting the existence of such a wave. The spectral methodology [12] was used to study the response when a crack front, moving as a straight line across $y = 0$, with uniform rupture speed v_o, suddenly encounters a localized tough region, represented by the black dot in the upper section of Figure 1. Within that region, G_{crit} was taken to be modestly higher, by ΔG_{crit}, than its uniform value $G_{crit,o}$ prevailing elsewhere. (For the calculation shown, $v_o \approx 0.5 c_R$ and $\Delta G_{crit} \approx 0.1 G_{crit,o}$ due to increase of both σ_o and δ_o by 5% within the tough region. That region has diameter $0.002\,\lambda$ where λ is a periodic repeat distance along the z axis, parallel to the crack front. Because of the spectral periodicity in the displacement basis set for the numerical formulation, the problem actually solved is that of an initially straight crack front which simultaneously impinges on a row of such tough regions that are spaced at distance λ along the z axis.)

The resulting perturbation, $v(z,t) - v_o$, in the crack front propagation velocity is shown in the upper part of Figure 1, replotted here from [6] to show the perturbation of velocity, rather than that of G_{rest} as in the original presentation. The perturbation seemed to propagate as a persistent wave, for as long as it was feasible to do the numerical calculation. The dashed lines show where *p, s* and *R* waves would intersect the moving crack front, so that the crack front wave speed c_f, measured relative to the position of the tough region where it nucleated, is seen to be a high fraction of c_R. The motivation in [6,11] to examine the response to such a small localized perturbation arose from understanding of the scalar model [7], for which the form of response to an isolated excitation was critical to understanding the response to sustained (small) random excitation [8] generated by a spatial fluctuation of $\Delta G_{crit} = \Delta G_{crit}(x,z)$ along the crack plane.

For the singular model, Ramanathan and Fisher [10] further developed the Willis-Movchan [9] analysis and showed that the equations of elastodynamics do indeed imply a persistent wave, at least when G_{crit} is independent of crack speed *v*. They also noted that increase of G_{crit} with *v* damps the wave (viscoelastic properties of the material would be expected to do also); that has not yet been fully quantified. Further, they showed that the speed is a high fraction of c_R , consistent with the wave as seen in the simulation. Figure 2 shows results [13], obtained by evaluating the analytically determined expression [10], for the crack-front-parallel wave speed, $\sqrt{c_f^2 - v_o^2}$, as a function of crack speed v_o for different values of the Poisson ratio ν. It is seen that typically $\sqrt{c_f^2 - v_o^2} \approx 0.96\sqrt{c_R^2 - v_o^2}$ at modest speeds, thus showing that $c_f \approx 0.96 c_R \sqrt{1 + 0.1 v_o^2 / c_R^2}$.

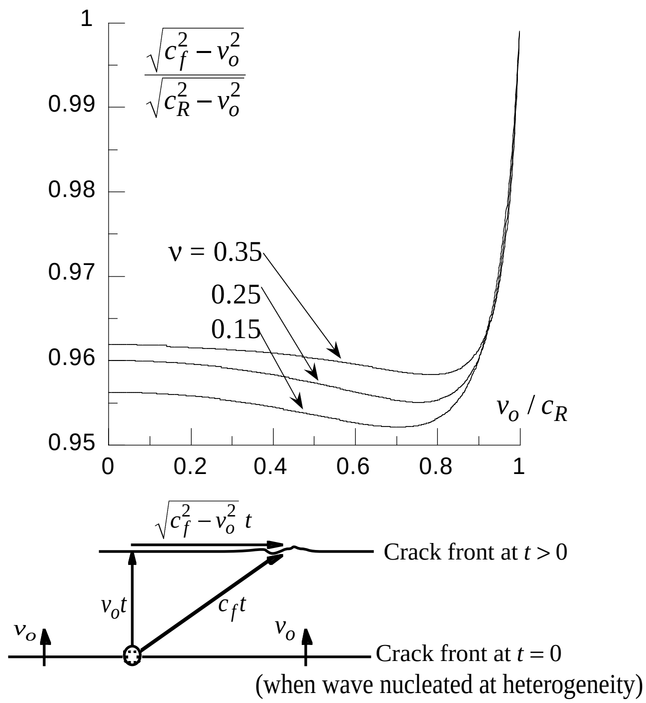

Figure 2 Crack front wave speed, from [13] based on solution in [10].

Many aspects concerning these waves remain to be understood. These include damping due to $dG_{crit} / dv > 0$ and to material viscoelasticity and, most especially, to such generalizations of crack front waves as may exist when there is non-planarity of growth involved in perturbations of the crack front. E. g., such non-planarity must be a part of the Wallner line process [14] since those features are observed optically on what are, otherwise, mirror smooth surfaces.

3. Limiting Fracture Speed and Fracture Roughening

As explained above, according to the singular crack model, for ruptures along a plane we expect that under typical remote loading conditions v will accelerate towards the limiting speed c_R. However, to summarize observations [3,15,16]: v in brittle amorphous solids (glass, PMMA) has an upper limit of order 0.50 - 0.60 c_s (i.e., 0.55 - 0.65 c_R). The fracture surface is mirror-smooth only for $v < 0.3 - 0.4\,c_R$. The crack surface roughens severely, with profuse micro-crack formation off of the main crack plane, and v becomes strongly oscillatory, at higher (average) speeds. The crack forks at the highest speeds. There are exceptions for which v approaches c_R. These involve cases for which the fracture does remain confined to a plane: brittle single crystals and incompletely sintered solids [17].

Interferometric studies [15] of propagating fractures in PMMA sheets showed the irregularity of the propagation process as the terminal speed (substantially sub-c_R) was approached. Strong stress waves were radiated from the tip in distinctly separated pulses. Later studies [16] clearly tied the strongly intermittent v, and intensity of fluctuation of v about the mean, to a process which created micro-fractures whose path was side-branched off the main crack path. That is, small fractures seem to have emerged out of the walls of the main fracture near to its tip (like in Figure 3a) such that the main crack and one or a pair of such side-branching cracks coexisted for a time, although the branches were ultimately outrun by the main fracture, at least at low mean propagation speeds. The origin of those side-branched features is, however, disputed [18]. Rather than originating at the main crack surface, it has been argued [18] that the side branched cracks may instead have began their life as part, not well aligned with the main crack plane, of a damage cluster of microcracks developing ahead of the main crack tip, Figure 3b, which formed a coherent larger crack and grew into the main crack walls. Thus there are competing views, which remain to be resolved, of the origin of dynamic roughening.

In any event, the density of cracks left as side-branching damage features increases substantially with increase of the loading which drives the fracture, as does the surface roughening. Ultimately, the response of the fracture to further increase in loading (precisely, to increase in G_{rest}) is no longer to increase the average v, which is the result expected from the theoretical analysis of a crack moving on a plane, but rather to absorb more energy by increasing the density of cracks in the damage cluster formed at the tip of the macroscopic fracture. Thus the macroscopic fracture energy G_{crit} rises steeply with crack speed [16], even if a locally defined G_{crit} at the tip of each microcrack does not.

The deviation of the fracture process from a plane is tied yet more definitively to the intermittency and low limiting crack speeds by studies [17] of weakly sintered plates of PMMA. The resistance to fracturing along the joint was much smaller than for the adjoining material. That kept the crack confined to a plane, which led to smooth propagation of the fracture (no evidence of jaggedness of interferograms by stress wave emission), with v reaching 0.92 c_R.

The attempts to explain low limiting speeds and fracture surface roughening begin with the first paper, by Yoffe [19], in which the elastodynamic equations were solved for a moving crack. That was done for a crack which grew at one end and healed at the other, so that the field depended on $x - vt$ and y only (2D case). Nevertheless, it sufficed to reveal the structure of the near tip singular field, including the v dependence of the $\Sigma_{\alpha\beta}$ functions above. The hoop stress component $\sigma_{\theta\theta}$, at fixed small r within the singularity-dominated zone, reaches a maximum with respect to θ for $\theta \neq 0$ when $v > 0.65\,c_R$. That gives a plausible explanation of limiting speeds and macroscopic branching, but does not explain the side-branching and roughening which set in at much smaller speeds.

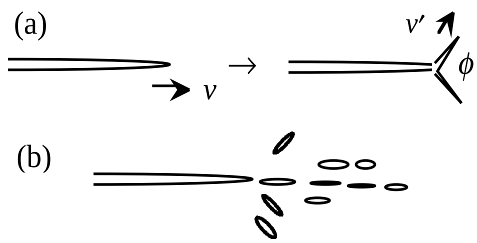

Figure 3. Two views of the origin of surface roughening.

Eshelby [20] took a different approach in posing the question of what is the v at which a fracture must be moving so that, by slowing down to a speed v', it can provide enough energy to drive two fractures. The extreme is to slow down to $v' = 0^+$. That question still has no precise answer when the angle ϕ (Figure 3a) is non-zero, but Eshelby answered it in the limit $\phi \to 0$ for mode III (the Freund [4] mode I solution was not yet available). That $\phi \to 0$ version reduces to the solved problem of sudden change of speed of a fracture moving along the plane. The condition is $g(v) < 1/2$ (assuming no v dependence of G_{crit}), which for mode III gave $v > 0.60\,c_s \approx 0.65\,c_R$. Freund [4] addressed the same $\phi \to 0$ limit for mode I; with re-calculation [21] that leads to $v > 0.53\,c_R$. By working out the static version of the $\phi \neq 0$ problem, Adda-Bedia and Sharon [21] optimize with respect to ϕ to estimate that the lowest v for branching is $\approx 0.50\,c_R$. Material response with $dG_{crit} / dv > 0$ will reduce that threshold but, at present, it seems unlikely that the inferred onset of roughening at speeds in the range 0.3 - 0.4 c_R can be explained by a branching instability at a crack tip in a solid which is modeled, otherwise, as a linear elastic continuum. That argues that perhaps the scenario sketched in Figure 3a should be replaced by another, possibly like that discussed in connection with Figure 3b.

Nevertheless, it is interesting that some discrete numerical models of cracking do give surface roughening, and in some cases side-branching from the tip like in Figure 3a, at roughly realistic speeds. A transition to zig-zag growth was shown to set in around

$0.33\,c_R$ in molecular dynamics simulations [22]. It is not yet clear how the v at its onset, or whether roughening sets in at all, depends on details of the force law. Cohesive finite elements [23], or cohesive element interfaces [24-27] that fully model the separation process allow, within constraints of the mesh, for self-chosen fracture paths. They have shown off-plane fracturing by a process similar to what is shown in Figure 3a. In particular, Xu and Needleman [24] reproduce the observations that when the crack is confined to a weak plane, it accelerates towards c_R, whereas for cracking in a uniform material, side branches form at $0.45\,c_R$ or at a realistic slightly lower speed [25] if statistical variation in cohesive properties is introduced.

Recent work [26] suggests that such propensity for low-speed side branching is not universal for all cohesive finite element models. In particular, models which involve no opening or sliding at element boundaries until a strength threshold is reached [27], after which there is displacement-weakening (much like for the inset in Figure 1), have so far not shown a side-branching process analogous to Figure 3a or to the results of [24,25]. Rather, the fractures remain planar, or nearly so, and accelerate towards c_R [26]. It now seems important that the procedures of [24,25] allow opening and sliding displacements at the element boundaries before achieving the peak cohesive strength and that there is substantial non-linearity as the peak strength is approached. That is in contrast with the models in which the element boundaries remain unseparated up to peak strength [26], and for which similar side-branching has so far not been seen. The origin of this sensitivity to details of the cohesive formulation remain to be determined. The results may be confirming a proposal that local nonlinear features of the pre-peak deformation response are critical to understanding the onset of roughening [28].

Nearly all the theoretical work on crack roughening has been carried out in the context of a process like in Figure 3a, in which conditions for a branching or other instability are sought at the tip of the main fracture. More attention would be appropriate to understanding the effects of the opening, and sometime joining together, of multiple microcracks or microcavities ahead of the main fracture [18], like in Figure 3b. That is likely to be an important case not only for brittle amorphous solids under dynamic stressing as discussed here, but also for surface roughening in broad classes of materials [29], whether subjected to quasi-static or dynamic failure conditions.

4. Acknowledgment

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