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CONSTRAINTS ON THE DIFFUSIVE CAVITATION OF ISOLATED GRAIN BOUNDARY FACETS IN CREEPING POLYCRYSTALS

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Abstract—Following concepts introduced by B. F. Dyson (*Metal Sci.* 349 1976), the diffusive cavitation of grain facets is considered in circumstances for which the cavitated facets are well separated from one another. In this case the requirement of geometric compatibility between the opening grain facets and the creep-deforming polycrystalline surroundings reduces the stress transmitted to the cavitated facets, and hence increases the rupture lifetime. An evaluation of the rupture time, t_r, based on diffusional cavity growth to coalescence shows that t_r is given by the sum of two terms, one proportional to $1/D\sigma_\infty$ (where D is the grain boundary diffusion parameter and σ_∞ the stress which would act on a non-cavitated facet) and another proportional to $1/\dot{E}_\infty$ (here $\dot{E}_\infty$ is the creep rate of a similarly loaded polycrystal with uncavitated boundaries). The latter term is found to be much larger than the first at sufficiently low stress and temperature, as long as the cavitated facets are indeed well separated. This circumstance leads to results in which the cavity growth process and strain to rupture are consistent with the diffusional mechanism, but in which the rupture time t_r follows a Monkman–Grant correlation with t_r proportional to $1/\dot{E}_\infty$.

Résumé—En suivant des idées introduites par B. F. Dyson (*Metal Sci.* 349 1976), nous avons étudié la cavitation intergranulaire par diffusion sur des facettes de grains, lorsque Ces facettes sont bien séparées les unes des autres. Dans ce cas, la compatibilité géométrique entre les facettes des grains qui s'ouvrent et le matériau polycristallin déformé en fluage qui les entoure réduit la contrainte transmise aux facettes à cavités et augmente donc la durée de vie à la rupture. Une évaluation du temps de vie à la rupture t_r, à partir d'une croissance des cavités par diffusion jusqu'à la coalescence montre que t_r est la somme de deux termes: le premier est proportionnel à $1/D\sigma_\infty$ (où D est le paramètre de diffusion intergranulaire et σ_∞ est la contrainte que agirait sur une facette sans cavité) et le second est proportionnel à $1/\dot{E}_\infty$ (où $\dot{E}_\infty$ est la vitesse de fluage d'un polycristal sans cavité soumis à la même charge). Le second terme est beaucoup plus grand que le premier pour des contraintes et des températures suffisamment basses, tant que les facettes à cavités sont bien séparées. Ceci conduit à un processus de croissance des cavités et à une déformation à la rupture compatibles avec un modèle de diffusion, alors que le temps de rupture t_r obéit à une corrélation de Monkman et Grant, avec t_r proportionnel à $1/\dot{E}_\infty$.

Zusammenfassung—Die Hohlraumbildung an Korngrenzfacetten über Diffusionsprozesse wurde für den Fall, daß die Facetten weit voneinander entfernt sind, nach den von B. F. Dyson eingeführten Konzepten (*Metal Sci.* 349, 1976) untersucht. In diesem Fall muß zwischen den sich öffnenden Facetten und der kriech-verformten Umgebung geometrische Kompatibilität herrschen, welches die auf die Hohlraumbesetzten Facetten übertragene Spannung reduziert. Dadurch wird die Bruchstandfestigkeit vergrößert. Wertet man auf der Grundlage des Diffusions-kontrollierten Zusammenwachsen von Hohlräumen die Zeit t_r bis zum Bruch aus, so zeigt sich, daß t_r aus der Summe zweier Terme besteht: einem proportional zu $1/D\sigma_\infty$ (D: Parameter der Korngrenzdiffusion, σ_∞: die auf eine Facette ohne Hohlräume wirkende Spannung) und einem proportional zu $1/E_\infty$ (E_∞: Kriechrate eines vergleichbar belasteten Polykristalles ohne Hohlräume an den Korngrenzen). Dieser $\dot{E}_\infty$-Term ist bei genügend niedrigen Spannungen und Temperaturen viel größer als der andere Term, solange die Facetten mit Hohlräumen wirklich getrennt sind. Diese Umstände führen zu Ergebnissen, bei denen der Prozess des Hohlraumwachstums und die Bruchdehnung mit einem Diffusions-bestimmten Mechanismus verträglich sind, bei dem jedoch die Zeit t_r bis zum Bruch einer Monkman–Grant-Korrelation mit t_r proportional zu $1/\dot{E}_\infty$ folgt.

INTRODUCTION

There is an extensive literature on the diffusive growth of grain boundary cavities in elevated temperature creep rupture. The basic model for the process was developed by Hull and Rimmer [1] and improved in various ways by subsequent workers [2]; results of such studies have been reviewed recently, e.g., in Ref. [3] where extensions of the model to include nonequilibrium cavity shapes are discussed, and in Ref. [4] which deals additionally with the coupling of dislocation creep to the diffusive growth process.

The result of such studies is that the cavity growth rate has been related to cavity size and spacing and to the average stress, σ, acting over the cavitated grain facet. However, in applications of the theoretical relation, it has usually been tacitly assumed that σ could be equated to the macroscopic stress (say, σ_∞)

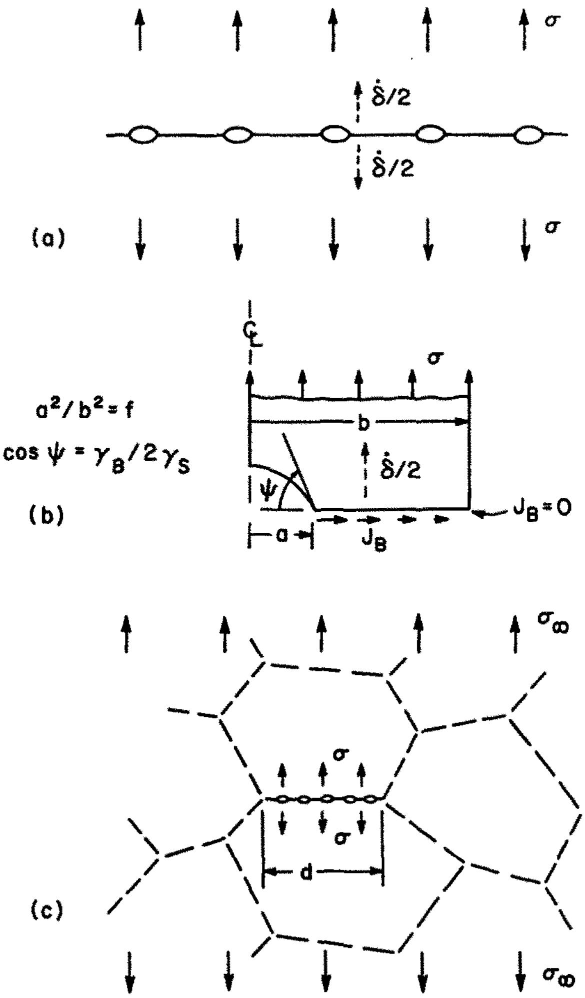

Fig. 1. (a) A cavitated grain boundary, subjected to stress σ and separating at rate $\dot{\delta}$ due to diffusional flow of matter from the cavity surfaces into the grain boundary. (b) Axi-symmetric geometry employed for analysis of cavity growth; f is the area fraction of cavitated boundary. (c) An isolated, cavitated grain boundary facet in a polycrystal; the stress σ will generally be reduced from the stress σ_∞ which would act across the facet in the absence of cavitation.

acting on a polycrystal. In an important paper, Dyson [5] questions the reasonableness of this assumption. He observes that cavitated grain boundary facets are often relatively isolated from one another and, in this circumstance, the rate at which openings δ develop across the cavitated facets must be compatible with the rate at which creep deformations of the surrounding material can accommodate such openings. The cavitated facets must then generally shed load to the surroundings until the stress σ that they carry is reduced sufficiently relative to σ_∞ that the opening rate $\dot{\delta}$ is compatible with the deformation rate of the surroundings. The result of the reduction in σ is a reduction of the rate of the diffusive cavity growth process and, in the limit of very low overall creep rates, the rate of cavity growth is determined entirely by the overall creep rate and not by the kinetics of the diffusive cavitation process.

This limiting situation, in which the cavity growth rate is proportional to the overall creep rate of essentially uncavitated material (and denoted as $\dot{E}_\infty$ here) is compatible with a Monkman–Grant [6] correlation (i.e., $\dot{E}_\infty t_r$ approximately constant, where t_r is the rupture time). Curiously, the theoretical basis for such a correlation has been sought recently [4, 7, 8] by concentrating on an opposite limiting case, in which the overall dislocation creep rate is large enough to interact significantly with the mechanism of diffusive matter transport. A fairly definitive analysis [4] of this coupled process suggests, however, that predicted times for cavity growth to coalescence (based again on the tacit assumption that $\sigma = \sigma_\infty$) lead to a Monkman–Grant product that increases with stress and temperature over the practical range of each in creep applications.

This paper is an attempt to quantify Dyson's concepts in a simple but approximate manner, so that the rate of cavity growth and rupture time can be predicted.

Analysis of constrained cavity growth

Figure 1a shows the configuration which has been analyzed for void growth. Details of the calculation are done by considering the geometry of Fig. 1b, where a spherical caps void of radius a is centered in a right-circular cylinder bi-crystal of radius b; b is chosen so that $a^2/b^2 = f =$ area fraction of grain boundary which is cavitated. The applied stress is σ, as shown, and the problem is analyzed subject to a boundary condition that J_B, the grain boundary diffusive flux, vanishes at the outer radius of the grain boundary in Fig. 1b.

When σ is sufficiently small, it is appropriate to assume that the void retains a quasi-equilibrium spherical caps shape [3] and that deformability of the adjoining crystals can be neglected [4] (in the sense that the separation velocity $\dot{\delta}$ can be regarded as being the same at all points along the grain interface). In this case the expression for the rate of cavity growth can be taken from Ref. [4], which incorporates some corrections of previously published results. The result for the rate of increase of volume V of an individual void is

$$\dot{V} = 4\pi D[\sigma - (1-f)\sigma_0]/[\ln(1/f) - (3-f)(1-f)/2]. \quad (1)$$

Here

$$D = D_B\delta_B\Omega/kT$$

is the grain boundary diffusion parameter ($D_B\delta_B$ = boundary diffusivity, Ω = atomic volume, kT = energy measure of temperature), and

$$\sigma_0 = 2\gamma_s(\sin\psi)/a$$

is the "sintering" stress (γ_s = surface free energy, ψ = cavity tip angle, Fig. 1b). Two related parameters of interest are calculated from equation (1). First, by writing the cavity volume as

$$V = (4\pi/3)a^3h \quad (2)$$

where [3] $h = h(\psi)$ depends on the cavity tip angle (and $h = 0.6$ for $\psi = 70°$, as is typical), one has

$$\dot{a} = \dot{V}/4\pi a^2h = (D/a^2h)[\sigma - (1-f)\sigma_0]/[\ln(1/f) - (3-f)(1-f)/2] \quad (3)$$

for the rate of cavity growth. Also, for mass conservation it is evident from the geometry of Fig. 1b that

$$\dot{V} = \pi b^2\dot{\delta},$$

so that the rate of opening of the cavitated boundary is

$$\dot{\delta} = \dot{V}/\pi b^2 = (4D/b^2)[\sigma - (1-f)\sigma_0]/[\ln(1/f) - (3-f)(1-f)/2]. \quad (4)$$

Now, as noted in the discussion based on Dyson's [5] paper, for a relatively isolated, cavitated grain facet (Fig. 1c), the stress σ and opening rate $\dot{\delta}$ in equation (4) must be consistent with the constraints set by the surroundings of that facet. The deformation of the surroundings should be modelled in a way dependent, for example, on the proximity of other cavitated facets, on the ease of sliding of grain boundaries, and on the deformation mode (dislocation and/or diffusional creep) of the grains. In the interest of developing a simple approximation, we shall consider σ and $\dot{\delta}$ to represent average quantities for the facet and neglect the fact that both of these will be nonuniform.

If the grain facet of diameter d (Fig. 1c) is modelled as an isolated disk-shaped crack in a homogeneous, linear viscous material, subjected to a remotely applied uniaxial tension σ_∞ (causing remote strain rate $\dot{E}_\infty$) and to a tension σ on the crack surfaces, restraining them against opening, then the average opening rate of the grain facet is given by (see Appendix)

$$\dot{\delta} = \alpha[(\sigma_\infty - \sigma)/\sigma_\infty]\dot{E}_\infty d, \quad (5)$$

where α is a dimensionless factor. It is shown in the Appendix that $\alpha = 2/\pi$ for the linear viscous material. For a non-linear but still homogenous viscous material, with $\dot{E}_\infty \propto \sigma_\infty^n$ as frequently used to model power-law creep, we assume that $\dot{\delta}$ is still given by an expression in the form of equation (5) but with a different α. It is easy to see from dimensional considerations that α can depend only on n and on σ/σ_∞, and as an approximation it is assumed that the stress dependence of the result is given by the bracketed term in equation (5) so that α depends only on n. Noting that $\dot{\delta}\pi d^2/4$ is the volumetric opening rate of the penny shaped crack, we can estimate approximately the effects of n on α by using recent results of Budianksy *et al.* [9] for the volumetric opening rate of a spherical hole in a nonlinear viscous material. (Note that expressions for the volumetric opening rate of a spherical hole and of a narrow disc-shaped void of the same diameter are very close in the case of a linear viscous material in uniaxial tension; the spherical hole opens at a rate which is just $\pi/4$ times that for the disc-shaped void [4].) Taking the ratio of the volumetric growth rate from Budiansky *et al.* [9] for a void in a nonlinear viscous material with $n = 5$ to that for a void in a linear material, and noting the form of equation (5), one estimates that

$$\alpha/\alpha_{\text{linear}} \approx 1.42 \text{ for } n = 5.$$

Thus, since $\alpha_{\text{linear}} = 2/\pi = 0.64$, the estimate is $\alpha = 0.90$ for $n = 5$. Corresponding results are $\alpha = 0.84$ for $n = 3$ and $\alpha = 0.98$ for $n = 10$. Hence, if power-law dislocation creep takes place among the adjoining grains, with negligible grain boundary sliding, a value of $\alpha = 0.9$ can be considered representative in equation (5).

Slightly different forms of the result in equation (5) result for other mechanisms. For example, if all grain boundaries slide freely, it is appropriate to interpret σ_∞ in equation (5) not as the remotely applied stress but rather as the stress which would result on a coherent, non-cavitating grain facet oriented perpendicular to the tensile stress. This is estimated in the Appendix as $\sigma_\infty \approx 1.5 \times$ (the remotely applied stress); the result applies regardless of whether the grains deform by linear or nonlinear creep or, indeed, of whether the grain interiors remain rigid and all macroscopic deformation occurs by diffusional volume and/or grain boundary transport. The presence of freely sliding grain boundaries does, however, increase α over the estimates given previously. The amount of increase is unknown but it is argued in the Appendix that perhaps a doubling of the previous α values (equivalent to doubling the effective size of the cavitated facet diameter, d) would provide a reasonable estimate, so long as the cavitated boundaries are indeed very well isolated from one another.

When the cavitated facets are not far enough spaced to be considered isolated, interactions occur which have the effect of further increasing α over the earlier estimates, at least if $\dot{E}_\infty$ is consistently interpreted in equation (5) as the strain rate of a similarly stressed but uncavitated polycrystal. A limiting case discussed by Dyson is that when all facets approximately perpendicular to the tensile direction are cavitated, and the grain boundaries slide freely. For this case α would be very large compared to unity which, as will be seen, has the effect of making $\sigma = \sigma_\infty$. Effectively, the concept of constraint by the surroundings does not apply in this case. At the present stage of this work there is no good estimate of α for the range in which the cavitated facets are close enough together to interact with one another.

Now, equations (4) and (5) are two different expressions for $\dot{\delta}$, and it is evident that σ must take on a value which makes these two expressions consistent with one another. Hence,

$$\alpha(\sigma_\infty - \sigma)\dot{E}_\infty d/\sigma_\infty = (4D/b^2)[\sigma - (1-f)\sigma_0]/[\ln(1/f) - (3-f)(1-f)/2]$$

from which it follows that

$$\frac{\sigma - (1-f)\sigma_0}{\sigma_\infty - (1-f)\sigma_0} = \frac{\ln(1/f) - (3-f)(1-f)/2}{(4L^3/\alpha b^2 d) + \ln(1/f) - (3-f)(1-f)/2} \quad (6)$$

Here

$$L = (D\sigma_\infty/\dot{E}_\infty)^{1/3} \quad (7)$$

is a stress level and temperature dependent parameter with length dimensions introduced by Rice [10]. Extensive tabulations of its values for pure metals undergoing power-law creep (with activation energy equal to that for self diffusion) have been given by Needleman and Rice [4]. The parameter has the form [4, 10]

$$L = L_0 \exp(\kappa T_m/T)(\mu/10^3\sigma_\infty)^{(n-1)/3}$$

where μ is the elastic shear modulus and L_0 and κ are tabulated constants. The results are such that at $T = 0.5T_m$ and $\sigma_\infty = 10^{-3}\mu$, $L \approx 2$–$6\ \mu$m for f.c.c. metals (but lower for Al and higher for Ag) and 0.25–0.35 μm for b.c.c. metals. The sizes for L increase by about a factor of 20 when σ_∞ is reduced by a factor of 10 to $10^{-4}\mu$; L also increases with decreasing temperature, in a manner consistent with $\kappa \approx 2.4$ to 3.9 for f.c.c. and 1.7–2.2 for b.c.c.

The significance of the parameter L as introduced in Refs [4] and [10] is that when L (but as based on the *local* stress σ and associated strain rate in the adjoining grains) is comparable to or smaller than the void half spacing length b, interactions between creep deformability of the grains and diffusion occur which invalidate equations (1), (3) and (4). When L is sufficiently large, such interactions do not occur. But it is seen from equation (6) that the constraint effects discussed here become important when L is large (more precisely, when $4L^3/\alpha b^2 d$ is comparable to or larger than $\ln(1/f)$) and σ may then be very much reduced from σ_∞, at least if the cavitated facets are well isolated.

The result for $\sigma - (1-f)\sigma_0$ from equation (6) may by used in equation (3) for $\dot{a}$ to obtain

$$\dot{a} = (D/a^2h)[\sigma_\infty - (1-f)\sigma_0]/[(4L^3/\alpha b^2 d) + \ln(1/f) - (3-f)(1-f)/2]. \quad (8)$$

This expresses the cavity growth rate under constrained conditions. Comparing to equation (3), it is seen that σ_∞ replaces σ and the numerator has been augmented by $4L^3/\alpha b^2 d$.

Expression for the rupture time

Using the previous result for $\dot{a}$, the "rupture time" t_r is calculated here. This time is defined as that for cavities to grow from some initial radius a_i to coalescence ($a = b$) on the cavitated facet. It should agree with the actual lifetime if the time to cavity nucleation is insignificant and if final failure follows shortly after complete cavitation of the isolated grain boundary facets. Otherwise, t_r might be considered a lower bound, although it is well to remember that the rupture process may be rather more complicated than this simple description suggests. For example, the shedding of load from the cavitated facets, which becomes more severe as f increases towards unity (see equation (6)), implies an increase of stress in adjoining material. This stress concentration may locally accelerate the grain boundary sliding rates and cause cavities to nucleate and grow on nearby facets which are favorably oriented relative to the tensile direction.

To compute t_r as defined, we represent $\dot{a}$ of equations (8) symbolically as $\dot{a} = 1/F(a)$ so that the rupture time is given as

$$t_r = \int_{a_i}^{b} F(a)\,\mathrm{d}a \qquad (9)$$

where a_i is the initial cavity radius. Now, by inspection it is clear that $F(a)$ can be split into the sum of two terms

$$F(a) = F_1(a) + F_2(a) \qquad (10)$$

where

$$\begin{aligned} F_1(a) &= (ha^2/D\sigma_\infty)[\ln(1/f) - (3 - f) \\ &\quad \times (1 - f)/2]/[1 - (1 - f)2\gamma_s \sin\psi/\sigma_\infty a] \\ F_2(a) &= (ha^2/D\sigma_\infty)(4L^3/\alpha b^2 d)/[1 - (1 - f)2\gamma_s \\ &\quad \times \sin\psi/\sigma_\infty a] \\ &= (4ha^2/\alpha\dot{E}_\infty b^2 d)/[1 - (1 - f)2\gamma_s \sin\psi/\sigma_\infty a] \end{aligned} \qquad (11)$$

and where it is recalled that $f = a^2/b^2$. The first of these, $F_1(a)$, is the form taken by $F(a)$ in the unconstrained case, when $\sigma = \sigma_\infty$ throughout the growth process. Accordingly,

$$t_r = (t_r)_1 + (t_r)_2 \qquad (12)$$

where

$$(t_r)_1 = \int_{a_i}^{b} F_1(a)\,\mathrm{d}a \qquad (13)$$

is the rupture time as predicted on the basis of the Hull–Rimmer growth mechanism for the unconstrained case of an infinite g·b (Fig. 1a) with $\sigma = \sigma_\infty$, and where

$$(t_r)_2 = \int_{a_i}^{b} F_2(a)\,\mathrm{d}a \qquad (14)$$

includes the effect of constraints on the growth mechanism. Note that these times scale with the parameters of the problem as

$$\begin{aligned} (t_r)_1 &= (hb^3/D\sigma_\infty) \\ &\quad \times (\text{a function of } a_i/b \text{ and } 2\gamma_s \sin\psi/\sigma_\infty a_i) \\ (t_r)_2 &= (hb/\alpha\dot{E}_\infty d) \\ &\quad \times (\text{another function of } a_i/b \text{ and } 2\gamma_s \sin\psi/\sigma_\infty a_i). \end{aligned} \qquad (15)$$

It is remarkable that $(t_r)_1$ contains only terms that refer to grain boundary diffusion and $(t_r)_2$ to the deformations of the constraining surroundings.

The last paramter, $2\gamma_s \sin\psi/\sigma_\infty a_i$, is the ratio of the sintering stress level at the start of growth to the remotely applied stress. This is often quite small compared to unity and when it is the integrals involved in equations (13, 14) are elementary, resulting in

$$\begin{aligned} (t_r)_1 &= \frac{16hb^3}{315D\sigma_\infty}\left[1 - \frac{105}{16} f_i^{3/2} \ln(1/f_i) \right. \\ &\quad \left. - \tfrac{1}{8} f_i^{3/2}\left(63 f_i^{1/2} - \frac{175}{4} - \frac{45}{4} f_i\right)\right], \\ (t_r)_2 &= \frac{4hb}{3\alpha\dot{E}_\infty d}(1 - f_i^{3/2}), \end{aligned} \qquad (16)$$

where

$$f_i = a_i^2/b^2.$$

It is instructive to consider the ratio of these two contributions to t_r. First consider the case when a_i/b is sufficiently small (say, 1/10 or less) that f_i can be replaced by zero in equations (16). In that case

$$\begin{aligned} (t_r)_2/(t_r)_1 &= 105D\sigma_\infty/4\alpha\dot{E}_\infty b^2 d \\ &= (105/4\alpha)(L^3/b^2 d) \end{aligned} \qquad (17)$$

where L is the temperature and stress-level dependent parameter of equation (7). Obviously, if the stress and temperature are sufficiently low that L^3 is of the order of $b^2 d$ or larger, and if the cavitated facets are sufficiently isolated that α is of the order of unity, then $(t_r)_2$ is many times larger than $(t_r)_1$. The ratio $(t_r)_2/(t_r)_1$ also increases with increasing a_i/b; for values of a_i/b equal to 1/5, 1/3 and 1/2 the ratio is, respectively, 1.15, 1.67 and 8.92 times the result for very small a_i/b in equation (17).

The Monkman–Grant product $\dot{E}_\infty t_r$ can be written as

$$\begin{aligned} \dot{E}_\infty t_r &= (4h/3\alpha)(b/d)(1 - f_i^{3/2})[1 + (t_r)_1/(t_r)_2] \\ &= (4h/3\alpha)(b/d)(1 - f_i^{3/2})\{1 + (4\alpha b^2 d/105L^3) \\ &\quad \times [G(f_i)/(1 - f_i^{3/2})]\} \end{aligned} \qquad (18)$$

where the function $G(f_i)$ represents the bracketed expression in the first of equation (16). Hence, in the circumstances just discussed when $(t_r)_1/(t_r)_2$ is small compared to unity,

$$\dot{E}_\infty t_r \approx (4h/3\alpha)(b/d)(1 - f_i^{3/2}). \qquad (19)$$

Typically, $h \approx 0.6$. Also, if the cavitated facets are indeed well isolated from one another, then it has been estimated that $\alpha = 0.9$ for power-law dislocation creep without grain boundary sliding and that α might increase to 1.8 for freely sliding boundaries. Hence in this case the factor in equations (18) and (19) has the range

$$4h/3\alpha \approx 0.44 \text{ to } 0.89$$

CONCLUSIONS

The problem of diffusive void growth on isolated grain boundary facets has been analyzed. It is shown that the "rupture time", as defined in the previous section, can be split as in equation (12) into the sum of two contributions, $(t_r)_1$ and $(t_r)_2$. As indicated by equations (15) and (16), $(t_r)_1$ is proportional to $b^3/D\sigma_\infty$ where $2b$ is the cavity spacing, D is the grain boundary diffusion parameter (defined following equation (1)), and where σ_∞ is either the applied stress or a factor of approximately 1.5 times it, depending on whether the grain boundaries are nonslipping or freely sliding; $(t_r)_2$ is proportional to $b/\alpha\dot{E}_\infty d$ where d is the grain facet size, $\dot{E}_\infty$ the creep rate of a similarly loaded but noncavitated polycrystal, and α is a parameter which has been estimated to vary from approximately 0.9–1.9 (depending again on whether the *g.b.*'s are nonslipping or freely sliding) for well isolated, noninteracting, cavitated facets, but which can be very much larger for interacting facets.

The ratio of $(t_r)_2$ to $(t_r)_1$ is approximately $(105/4\alpha)(L^3/b^2d)$, at least for small initial radii of the cavities and applied stress levels which are well above the sintering limit. Here L is the length parameter of equation (7) and Refs [4, 10]; it decreases with increasing temperature and stress in the power-law creep regime. This ratio of $(t_r)_2$ to $(t_r)_1$ can be very much larger than unity at sufficiently low stress and temperature. In such circumstances the Monkman–Grant product $\dot{E}_\infty t_r$ is predicted to have a constant value, equal approximately to $0.8b/\alpha d$, which is of the order 0.4 to 0.9 b/d for well isolated, noninteracting cavitated facets.

The paper shows, following concepts introduced by Dyson [5], that the void growth mechanism may be diffusional, and that rupture strains may have small values usually associated with diffusional growth, while the time to rupture is controlled by the overall dislocation creep rate.

It remains for future work to deal with cases for which the cavitated grain facets are sufficiently close to interact with one another [5], or for which the stress levels transmitted to the cavitated facets are large enough to invalidate the Hull–Rimmer diffusional growth model with a quasi-equilibrium, spherical-caps cavity shape [3] and with effectively rigid separation of the adjoining grains [4].

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APPENDIX

Consider a disc-shaped crack of diameter d in a homogeneous linear elastic solid subject to uniform tension σ_∞ far from the crack. The opening displacement ΔU between the upper and lower crack surfaces is available from many sources (e.g., Green and Zerna [11]) and is

$$\Delta U = [4\sigma_\infty(1 - \nu)/\pi G]\sqrt{(d/2)^2 - r^2}$$

where r is the distance from the crack center, ν the poisson ratio and G the shear modulus. By integrating ΔU over the area of the crack and writing the results as $\pi(d/2)^2\delta$, so that δ is the average opening, one obtains

$$\delta = 4(1 - \nu)\sigma_\infty d/3\pi G.$$

If the crack surfaces are not stress free, but instead transmit a tension σ then, by an elementary superposition

$$\delta = 4(1 - \nu)(\sigma_\infty - \sigma)d/3\pi G.$$

Now, since the remote strain E_∞ due to the remote uniaxial tension is $\sigma_\infty/2(1 + \nu)G$, the result may be rewritten as

$$\delta = [8(1 - \nu^2)/3\pi][(\sigma_\infty - \sigma)/\sigma_\infty]E_\infty d$$

By the analogy between linear elastic and linear viscous materials, the same result holds for a crack in a homogeneous linear viscous material if δ and E_∞ are replaced by their rates and, since the linear viscous material is usually regarded as incompressible, ν is replaced by 1/2. Hence one writes

$$\dot{\delta} = \alpha[(\sigma_\infty - \sigma)/\sigma_\infty]\dot{E}_\infty d,$$

as in equation (5) of this paper where $\alpha = 2/\pi$.

As mentioned in the paper, the same equation but with different α can be used approximately in other cases, When the material is inhomogeneous in the sense of having freely sliding grain boundaries, it is evident that σ_∞ should be interpreted as the value of σ when the opening rate $\dot{\delta}$ of the grain facet is zero. To obtain an approximate estimate of the relation of σ_∞ to the applied stress (say, σ_{appl}) in that case, suppose that the grain boundary array of Fig. 1c is changed into a regular two-dimensional array of hexagons and that there is no cavitation. In this case $\sigma = \sigma_\infty$ is to be interpreted as the average stress acting over the horizontal facet of width d in the figure. If one lets σ' denote the average stress in the same direction acting over the widest dimension of the hexagon, then overall equilibrium requires that

$$2\sigma' + \sigma_\infty = 3\sigma_{appl}$$

From symmetry considerations this widest horizontal line, as well as the vertical bisector of a hexagon, are acted upon by zero shear stress. Hence if one isolates a quarter of a hexagon, bounded by such horizontal and vertical lines, as a free body and sums forces parallel to the inclined grain boundary (which can transmit no shear stress) one obtains

$$2\sigma' = \sigma_\infty$$

Hence, by simultaneous solution, for uncavitated grain facets and freely slipping grain boundaries this regular hexagon model gives

$$\sigma = \sigma_\infty = (3/2)\sigma_{appl}, \quad \sigma' = (3/4)\sigma_{appl}$$

Hence σ_∞ in equation (5) is to be interpreted as approximately 1.5 σ_{appl} for freely slipping grains.

No analysis is available for estimation of the factor α in equation (5) for the freely slipping case. However, a simple approach suggested by the geometry of the regular hexagon model is to choose an effective facet width equal to d plus the projected width of the inclined facets, which gives a total effective width of $2d$, and to use the values of α as estimated for homogeneous materials. This is equivalent to using equation (5) with values of α which are twice as large as those estimated for homogeneous materials.