Presentation at Department of Energy Workshop on Time-Dependent Fracture of Materials at Elevated Temperature, Germantown, Md., 15-16 February 1979:

PLASTIC CREEP FLOW PROCESSES IN FRACTURE AT ELEVATED TEMPERATURES

James R. Rice
Division of Engineering, Brown University, Providence, R.I.

February 1979

Summary

This paper discusses recent theoretical developments on fracture at elevated temperature in the presence of overall plastic (dislocation) creep.

Two topics are considered:

1. Stress fields at tips of macroscopic cracks in creeping solids: Here consideration is given to the transient development of an effectively steady state of creep in a cracked body, following sudden load application for which the short-time material response is elastic. At long times (steady creep state) the severity of the near-crack-tip deformation rate field can be characterized in terms of a path-independent integral C^*, which is a generalization for non-linearly viscous materials of the J integral for rate-independent materials. Based on recent work by Riedel and Rice, it is shown that the short time transient field of creep flow has the same functional form at the crack tip, but that its amplitude parameter C^* is replaced, approximately, by $G/(1+n)t$, where t is time since load application, n is the exponent in a power-law creep relation $\dot{\varepsilon} \propto \sigma^n$, and G is Irwin's elastic energy release rate (calculated in terms of the stress intensity K_I as if the body were elastic). Hence $G/(1+n)C^*$ can be identified as a characteristic time for stress redistribution in attaining the steady creep state. Macroscopic creep crack growth is discussed in terms of these concepts and associated analyses.

2. Diffusive growth of microscopic grain boundary cavities in creeping solids: Previous analyses of this problem are based on an assumption that grains adjoining the cavitating boundary separate in an effectively rigid manner. However, important interactions between cavitation, by surface and grain boundary diffusion, and plastic creep processes are observed to occur when a state of overall dislocation creep prevails. These arise from two

effects. First, even in the absence of matter transport along the grain boundary (D_b very small), the presence of overall creep causes an increase in volume of the cavity and tends to cause a change in shape. The latter would, generally, tend to decrease rather than increase the cavity radius, but in the presence of sufficiently rapid surface diffusion the spherical-caps shape of the cavity is retained and the creep-flow-induced volumetric opening rate of the cavity causes a continuous enlargement of cavity radius. Second, when grain boundary diffusion is considered, the deformability of the adjoining grains means that matter diffusion from the cavity surfaces can be accommodated by highly localized relative separation velocities across the grain boundary. Hence the diffusion path length is not set by cavity spacing (as in the rigid-grain Hull-Rimmer model), but can be much shorter, resulting in a far more rapid removal of material from the cavity walls. A precise analysis of the problem has not yet been developed, but a variational principle governing simultaneous dislocation creep and grain boundary diffusion has been established, and this should lead to effective finite-element solution procedures. Also, an approximate model of the process suggests that for pure metals in the range of 0.5 to 0.8 Tm , and at stress levels of $10^{-3}\mu$ (μ = shear modulus) and $10^{-4}\mu$, respectively, the resulting cavity growth rate may exceed predictions of the rigid grain model by as much as factors of 10 to 100.

Stress fields at tips of macroscopic cracks in creeping solids:

This section is based on recent work of Riedel and Rice [1] on tensile (Mode I) cracks in elastic-creeping solids and on earlier work by Riedel [2] on anti-plane (Mode III) shear cracks. The materials considered are assumed to follow the stress strain relation

$$\dot{\varepsilon} = \dot{\sigma}/E + B\sigma^n \qquad (1)$$

in uniaxial tension. The exponent n is typically in the range 4 to 6, sometimes higher, for dislocation creep processes.

A body containing a crack is supposed to be loaded in tension. If the load is applied suddenly, the instantaneous stress field developed in the material is elastic. The stress concentration at the crack tip causes a zone of rapid creep straining to develop there, effectively to alleviate the elastic $r^{-1/2}$ stress singularity. The early stages of this process may be described in analogy to analyses for rate-independent plastic materials. Accordingly, this (short-time) field, in which elastic strains are much greater than creep strains everywhere except within a small region at the crack tip, is referred to as "small scale yielding." The near tip solution in this regime is complicated, but the parameter which governs it is the (far field) elastic stress intensity factor K_I.

On the other hand, at long times after load application there is complete redistribution of stresses and subsequent response of the material under fixed load takes place as if the material were purely viscous, $\dot{\varepsilon} = B\sigma^n$. In such cases, referred to as "extensive yielding," the near tip stress and creep rate fields have an intensity characterized by C^*, where C^* is a path-independent integral [3].

In both the "small scale" and "extensive" yielding limits, and for intermediate cases, the r,θ form of the near tip field is controlled (for $n>1$) by the non-linear term in (1). Accordingly, the near-tip fields of stress and strain rate are of the same form as the fields of stress and strain in a rate-independent material with $\varepsilon \propto \sigma^n$. Such fields are given in [4,5], and referred to as "HRR" fields.

For example, in the case of extensive yielding the near tip stress field (in plane strain or in plane stress) is given by an expression of the form

$$\sigma_{ij} \to \left(\frac{C^*}{Br}\right)^{1/(1+n)} f_{ij}(\theta) \qquad \text{as } r \to 0 \qquad (2)$$

where the f_{ij} are dimensionless and appropriately normalized functions of θ, dependent also on n.

By dimensional considerations, and by the requirement that the loading be characterized only by K_I, the short time, or small scale yielding, solution has the form

$$\sigma_{ij} = (EBt)^{-1/(n-1)} g_{ij}\left[r/K_I^2(EBt)^{2/(n-1)}, \theta\right] \qquad (3)$$

at time t after load application. Here g_{ij} is dimensionless, dependent also on n and ν (Poisson ratio), and $g_{ij}[\rho,\theta]$ decays as $\rho^{-\frac{1}{2}}$ for large ρ. This stress field involves a near tip singularity in the same form as (2) except that C^* is replaced by a factor proportional to K_I^2/Et. Indeed, Riedel and Rice [1] give an approximate argument showing that, for small scale yielding,

$$C^* \text{ should be replaced by } G/(1+n)t \qquad (4)$$

in (2), where G ($=(1-\nu^2)K_I^2/E$ for plane strain) is Irwin's energy release rate. Their comparisons with exact numerical results [2] for the field analogous to (3) in Mode III suggest that the approximation of (4) is accurate to ±10% for $n\geq 4$.

On the basis of (3) and the stress-strain relationship, it is possible to define a "creep zone," somewhat arbitrarily, as the region where creep strains exceed elastic strains, both reduced to equivalent tensile strains. This zone has a size which increases in proportion to the parameter

$$K_I^2 (EBt)^{2/(n-1)} ,$$

and small scale yielding conditions may be assumed to prevail whenever the creep zone is small compared to characteristic lengths of the cracked body (e.g., crack length, uncracked ligament width). By (4), a transition time t_1 between the short-time, small scale yielding and long-time, extensive yielding cases may be defined by

$$t_1 = G/(1+n)C^* . \qquad (5)$$

For example, consider a short plane stress crack of length a in a large body under stress σ_∞, with associated creep strain rate $\dot{\varepsilon}_\infty^{cr}$. For this geometry, $G = \pi\sigma_\infty^2 a/E$ is well known, and we write it as

$$\int_0^a G\, da = (\sigma_\infty^2/2E)[(\pi/2)(\sqrt{2}\, a)^2] , \qquad (6)$$

which can be interpreted as the loss of strain energy, $\sigma_\infty^2/2E$, on crack introduction, from an "affected" area consisting of a semi-circle of radius $\sqrt{2}\,a$. A similar interpretation gives, approximately

$$\int_0^a C^* da \approx \left[\frac{n}{n+1}\,\sigma_\infty \dot{\varepsilon}_\infty^{cr}\right]\left[\frac{\pi}{2}\left(\sqrt{2}\,a\right)^2\right] \quad , \text{ or } \quad C^* \approx \frac{2n}{n+1}\,\sigma_\infty \dot{\varepsilon}_\infty^{cr} \pi a \quad . \tag{7}$$

Thus the characteristic time t_1, for transition from small scale to extensive yielding is, approximately,

$$t_1 \approx \frac{1}{2n}\,\frac{\sigma_\infty}{E\dot{\varepsilon}_\infty^{cr}} = \frac{1}{2nEB}\,\sigma_\infty^{-(n-1)} \quad , \tag{8}$$

and is shorter at high stress levels than at low.

Correlation of creep crack growth by K_I seems appropriate when advance takes place over a time scale much less than t_1, and by C^* for a time scale much greater than t_1. The analyses just discussed are for a stationary crack; it is known [6] that there must be a different type of singularity at a growing crack tip than that described by (2), or by its short-time version based on (4). In analogy with growing crack solutions for rate-independent plastic materials [7], however, it is expected that this different singularity, for which elastic and creep strains are of the same order, will be important only in a small inner core of the heavily crept zone near the crack tip, for the more ductile of materials. The point needs further elaboration.

Diffusive growth of microscopic grain boundary cavities in creeping solids:

The well known Hull-Rimmer [8] model for diffusive void growth along a grain interface is illustrated in fig. 1. First, surface diffusion is presumed to be rapid enough so that the void retains a quasi-equilibrium spherical-caps shape (see Chuang et al. [9] for a detailed analysis of conditions under which this assumption is valid, and solutions to a more general version of the Hull-Rimmer model in cases for which it is not). Second, the grains are assumed to be effectively rigid, so that the only way in which the voids can grow (e.g., by grain boundary diffusion, which is typically the most rapid matter-transport process) is by diffusion along the whole grain interface between voids, since the grains must separate uniformly. This diffusion is driven by the difference between the potential, per unit volume, $-\sigma_n$ (σ_n = normal stress) on the interface and $-2\gamma_s\kappa$ (κ = surface curvature) on the void, which is negative whenever the net stress on the unvoided portion of interface exceeds the sintering limit of $2\gamma_s \sin\psi/a$.

But at applied stresses of the order $10^{-3}\mu$ (μ = shear modulus) at $0.5\ T_m$ or, for example, $10^{-4}\mu$ at $0.8\ T_m$, plastic creep flow of the grains is generally rapid enough, according to the data summarized by Ashby in [10], that the grains can hardly be considered rigid. Besides, it is known on empirical grounds (Monkman-Grant correlation) that the product of rupture time t_r and steady state creep strain rate $\dot{\varepsilon}_{ss}$ is not strongly variable over variations of stress and temperature that cause changes by several powers of 10 in $\dot{\varepsilon}_{ss}$. This suggests a strong coupling between plastic creep flow and creep rupture, even though diffusive processes as envisioned in the Hull-Rimmer model seem to be active.

There are two major ways in which plastic creep flow can interact with diffusive matter transport processes, and the net effect seems typically to be a significant increase of the void growth rate over what is predicted for the rigid grains model. The first way is illustrated in fig. 2a where, for simplicity, it is assumed that there is negligible matter transport along the grain boundary. Since the grains flow in creep, the material points immediately adjacent to the void surface take on a distribution of velocities which tend to make the void increase in volume and, in general under uniaxial tension and for widely spaced voids (so that plastic flow is not concentrated in a voided layer adjoining the grain boundary), to make

the void radius, a, decrease. This change in size and shape is indicated schematically by the dashed curve in fig. 2a. But when surface diffusion is rapid, local matter transport along the void surface retains the spherical caps shape (dash-dot-dash curve in fig. 2a) so that the net effect is to increase the void radius. Hence, if $\dot{V}_{cr}$ is the rate of void volume enlargment due to creep flow on the adjoining grains, the contribution to the growth rate is

$$\frac{d}{dt}\left[\frac{4\pi}{3}a^3 h(\psi)\right] = \dot{V}_{cr} \quad , \quad \text{or} \quad h(\psi)\dot{a} = \dot{V}_{cr}/4\pi a^2 \ . \tag{9}$$

Here the bracketed term is the void volume; $h(90°) = 1$, $h(70°) = 0.61$ ($70°$ is a typical angle ψ for metals). $\dot{V}_{cr}$ may be evaluated from classical creeping flow solutions for widely spaced spherical voids in a linear viscous material under uniaxial tension; it is not very different for a penny shaped crack, suggesting only a mild dependence on ψ . The result is

$$h(\psi)\dot{a} = \frac{1}{2\sqrt{3}}\dot{\varepsilon}_\infty a \ . \tag{10}$$

For comparison, the rigid-grains model predicts a result which, for $b/a \geq 5$, reduces to approximately [8,9]

$$h(\psi)\dot{a} \approx \frac{\mathcal{D}(\sigma_\infty - 2\gamma_s \sin\psi/a)}{2a^2 \ell n(b/2.1a)} \quad , \quad \text{where} \quad \mathcal{D} = \frac{D_b \delta_b \Omega}{kT} \tag{11}$$

and the notation is standard. In fact the ratio $\dot{a}$ from (10) to that from (11) is typically of the order a^3/L^3 where L is a stress level and temperature dependent length defined by

$$L = (\mathcal{D}\sigma_\infty/\dot{\varepsilon}_\infty)^{1/3} \ . \tag{12}$$

This length will appear subsequently and some numerical values will be given.

The second process by which plastic creep flow interacts with diffusion is illustrated in fig. 2b. Now matter transport along the grain boundary, with matter deposition on the adjoining grains, is considered. One the left is shown the void at one instant and two straight lines have been inscribed on the grains parallel to the grain boundary. On the right the void and inscribed lines are shown after some amount of growth. Obviously, in the rigid grain model the lines remain straight and matter must be transported

along the entire grain boundary. But, as remarked first by Beere and Speight [11], with plastic creep flow of the grains the matter can be accommodated locally, resulting in the strongly non-uniform motion of the inscribed lines as shown. This means that the diffusive path length can be much shorter than in the rigid-grains model, depending on how deformable the grains actually are, and this is expected to result in a more rapid removal of matter from the cavity walls (i.e., higher $\dot{a}$) than for the rigid grains model.

The process has not yet been modelled in a convincing way. Beere and Speight [11] assume that the grains separate in an effectively rigid way under low stress in some shell of material adjoining the void, with plastic creep flow taking place outside of this shell. However, the picture of the inscribed lines in fig. 2b suggests, instead, severe creep distortions near the cavity boundary. Such problems of combined plastic creep flow and diffusion are amenable to finite element analysis, and are being studied currently [12].

The finite element method is formulated according to a variational principle which leads to the system of equations shown in fig. 3. Here an axisymmetric problem of a spherical caps void of radius $r=a$, in a cylinder or radius $r=b$ ($\cong$ void half-spacing) is shown. The principle is written in dimensionless form with $R=r/a$, $B=b/a$, $H=h/a$, $Z=z/a$, $V_i = v_i/\dot{\varepsilon}_\infty a$, and is $\delta F=0$ ($F=$min.) where F is the following functional of dimensionless velocities V_i and associated dimensionless strain rates $\dot{E}_{ij} = \dot{\varepsilon}_{ij}/\dot{\varepsilon}_\infty$:

$$F = \int_0^H \int_{R_o(Z)}^B \frac{n}{1+n} \left(\sqrt{\frac{2}{3}\dot{E}_{ij}\dot{E}_{ij}}\right)^{(1+n)/n} RdRdZ$$

$$- \int_0^B (V_Z)_{Z=H}\, RdR$$

$$+ \left(\frac{a}{L}\right)^3 \int_1^B \left[\int_R^B R(V_Z)_{Z=0}\, dR\right]^2 \frac{dR}{R}$$

$$+ \frac{2\gamma_s \sin\psi}{a\sigma_\infty} \int_1^B R(V_Z)_{Z=0}\, dR \quad , \qquad (13)$$

where it is understood that $(V_R)_{R=B} = -\frac{1}{2} B$.

The first two terms of (13) are those which appear in a classical variational principle for creeping solids; the last two refer to g.b. (grain boundary) diffusion. When $L >> a$ and b, only the last three terms differ sensibly from zero in the solution field (i.e., the behavior is rigid in this limit) and the classical Hull-Rimmer result, summarized in (11), is recovered. On the other hand, when $L << a$, only the first two terms can differ sensibly from zero, grain boundary diffusion is unimportant, and the prediction of the growth rate reduces to that of (9), or of (10) for a linearly viscous material. At intermediate values of L, e.g., a/L of order unity, the coupling between g.b. diffusion and plastic creep flow is important.

To estimate L one may write

$$\mathcal{D} = \frac{D_{bo}\delta_b\Omega}{kT} \exp\left(-\frac{Q_b}{RT}\right) \tag{14}$$

and, following Ashby [10] for dislocation creep,

$$\dot{\varepsilon} = A \frac{D_{vo}\,\mu\, b}{kT} \left(\frac{\sigma}{\mu}\right)^n \exp\left(-\frac{Q_v}{RT}\right) \tag{15}$$

where, in the last expression A is a constant, μ the shear modulus, the dislocation slip step and D_{vo}, Q_v refer to bulk diffusion. Accordingly, with some rearrangement one may write

$$L \equiv (\mathcal{D}\sigma/\dot{\varepsilon})^{1/3} = L_o \exp\left(\frac{\kappa T_m}{T}\right)\left(\frac{10^{-3}\mu}{\sigma}\right)^{(n-1)/3} \tag{16}$$

where T_m is the melting temperature. Using data for all material parameters in (14,15) from the Ashby tabulation [10], values of n, κ, and L_o are shown for several metals in Table 1. Also shown are values of L at stress level of $10^{-3}\mu$ at 0.5 T_m and 0.8 T_m. A tenfold decrease in stress, to $10^{-4}\mu$ would increase the values of L shown by about a factor of 20.

What emerges, then, is that a, typically growing through a range of from 1 to 10 μm, will generally be of a size comparable to L at stress levels of order $10^{-3}\mu$ at 0.5 T_m and $10^{-4}\mu$ at 0.8 T_m. In such cases coupling between creep flow and g.b. diffusion must be considered. At significantly lower stress levels, L is much larger than a and b, and rigid grains behavior applies, eq. (11). At significantly higher stress levels, L is much smaller than a, and growth is described by eq. (9)

which, it may be recalled, predicts a result of order $(a/L)^3$ times that of eq. (11).

A highly approximate model in which the continuum of fig. 3 has been replaced by a "shear plate" has been studied by the writer in unpublished work. The variational functional F analogous to (13), but now reduced to a one-dimensional functional, has been minimized on the class of grain boundary velocity fields

$$(V_Z)_{Z=0} = H \exp\left(\frac{R-1}{\eta}\right)$$

(i.e., H and η chosen to minimize F), where ηa is the decay distance and the outer radius b is taken as infinite.

Some results of this very approximate analysis are quoted here, in lieu of accurate finite-element results, unavailable at present. Consider the case where σ_∞ is large compared to the sintering level, $2\gamma_s \sin\psi/a$, and take n=5 . In this case the predicted growth rate, $\dot{a}$, in presence of simultaneous creep and g.b. diffusion, is about equal to that predicted by the rigid grains model (with $h/a = 10$) when $a/L \approx 0.03$. The growth rate is 3 times higher than the rigid grains prediction when $a/L \approx 0.3$, 20 times when $a/L \approx 1$, 70 times when $a/L \approx 3$, and 500 times for $a/L \approx 10$. Thus, whenever a/L exceeds, say 0.1, the rigid grains model must be considered too conservative. When a/L is larger than about 10, eqs. (9) or (10) can be used with reasonable accuracy. As is seen from Table 1, the intermediate range, $0.1 < a/L < 10$, will be encountered in many practical cases.

Additional Note: A related study by Edward and Ashby ["Intergranular Fracture during Power Law Creep," Acta Met, in press] comes to similar conclusions on the importance of plastic creep flow on diffusive cavitation.

Acknowledgement: This research was supported by DOE under Contract EY-76-S-02-3084 with Brown University.

References

1. H. Riedel and J. R. Rice, "Tensile cracks in creeping solids," to be presented at May 1979 ASTM Annual Symposium on Fracture Mechanics, Washington Univ., St. Louis.

2. H. Riedel, Z. Metallkinde 69, 1978, 755-760.

3. J. D. Landes and J. A. Begley, in Mechanics of Crack Growth, ASTM STP 590, 1976, 128-148.

4. J. R. Rice and G. F. Rosengren, J. Mech. Phys. Solids 16, 1968, 1-12.

5. J. W. Hutchinson, ibid., 13-31.

6. H. Riedel and H. Hui, research in progress at Brown and Harvard Universities.

7. J. R. Rice and E. P. Sorensen, J. Mech. Phys. Solids 26, 1978, 163-186.

8. D. Hull and D. E. Rimmer, Phil. Mag. 4, 1959, 673.

9. T-j. Chuang, K. I. Kagawa, J. R. Rice and L. Sills, Acta Met., in press, 1979.

10. M. F. Ashby, Acta Met. 20, 1972, 887-897.

11. W. Beere and M. V. Speight, Met. Sci., 1978, 172-176.

12. J. R. Rice and A. Needleman, research in progress at Brown University.

Figure Captions

Fig. 1 Hull-Rimmer model for grain boundary cavitation by surface and grain-boundary diffusion. The adjoining grains are assumed to separate as rigid bodies in this model.

Fig. 2 (a) Cavity growth by combination of plastic creep flow (which, if considered alone, generally tends to decrease cavity radius, dashed curve) and rapid surface diffusion.

(b) Local accommodation of matter diffused into grain boundary, by deformation of grains. Note that the inscribed (dashed) lines do not remain straight, as assumed in the rigid-grains model, and thus the diffusion path length necessary to accommodate a given amount of matter is shorter.

Fig. 3 Summary of field equations and boundary conditions for axisymmetric problem of combined plastic creep flow and grain boundary diffusion. Outer radius b of cylinder represents half-spacing between adjacent voids.

Table 1

Values of $L \equiv (\mathcal{D}\sigma/\dot{\varepsilon})^{1/3} = L_o e^{\kappa T_m/T}\left(\frac{\mu}{10^3\sigma}\right)^{(n-1)/3}$

Material	n	κ	L_o (μm)	L (μm) at 0.5 T_m	L (μm) at 0.8 T_m
Ag	5.3	3.08	2.87×10^{-2}	13.6	1.35
Cu	4.8	3.05	1.09×10^{-2}	4.85	.493
Ni	4.6	3.92	$.511 \times 10^{-2}$	13.0	.686
Al	4.4	2.57	$.762 \times 10^{-2}$	1.30	.189
γFe	5.75	2.44	$.955 \times 10^{-2}$	1.26	.202
Zn	6.1	1.79	2.02×10^{-2}	.724	.189
αFe	6.9	2.20	$.213 \times 10^{-2}$	.174	.0332
Mo	4.3	1.70	$.396 \times 10^{-2}$	1.18	.330

values are for $\sigma = 10^{-3}\mu$; would be approximately 20 times larger for $\sigma = 10^{-4}\mu$, if n=5 .

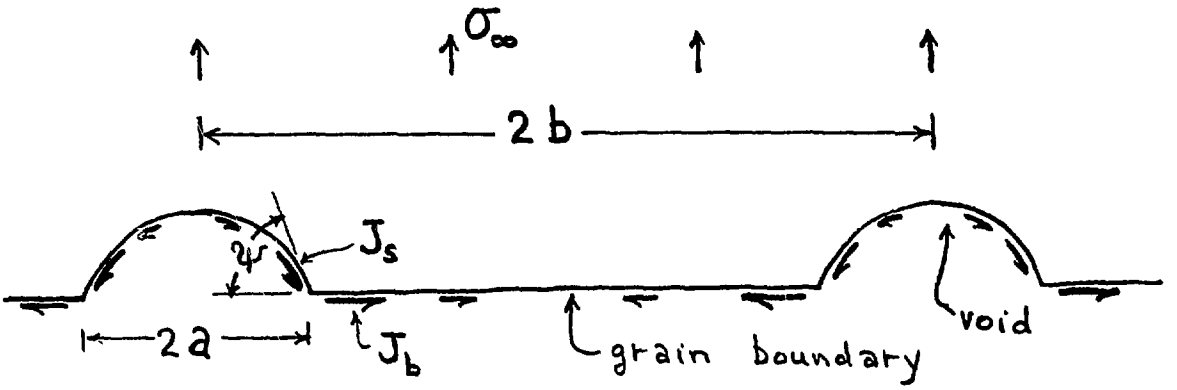
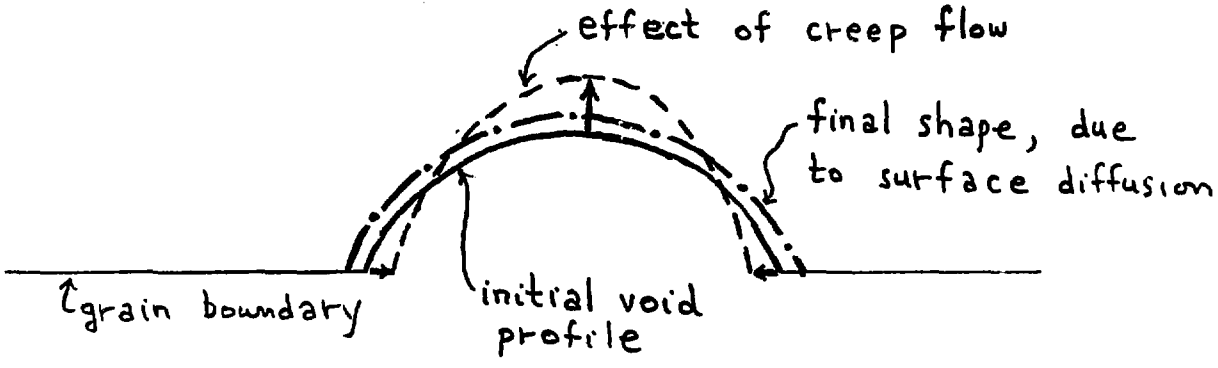


Fig. 1


effect of creep flow
final shape, due to surface diffusion
grain boundary
initial void profile
(a)

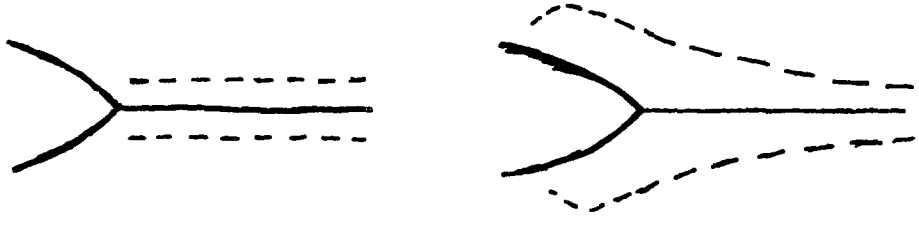

(b)

Fig. 2

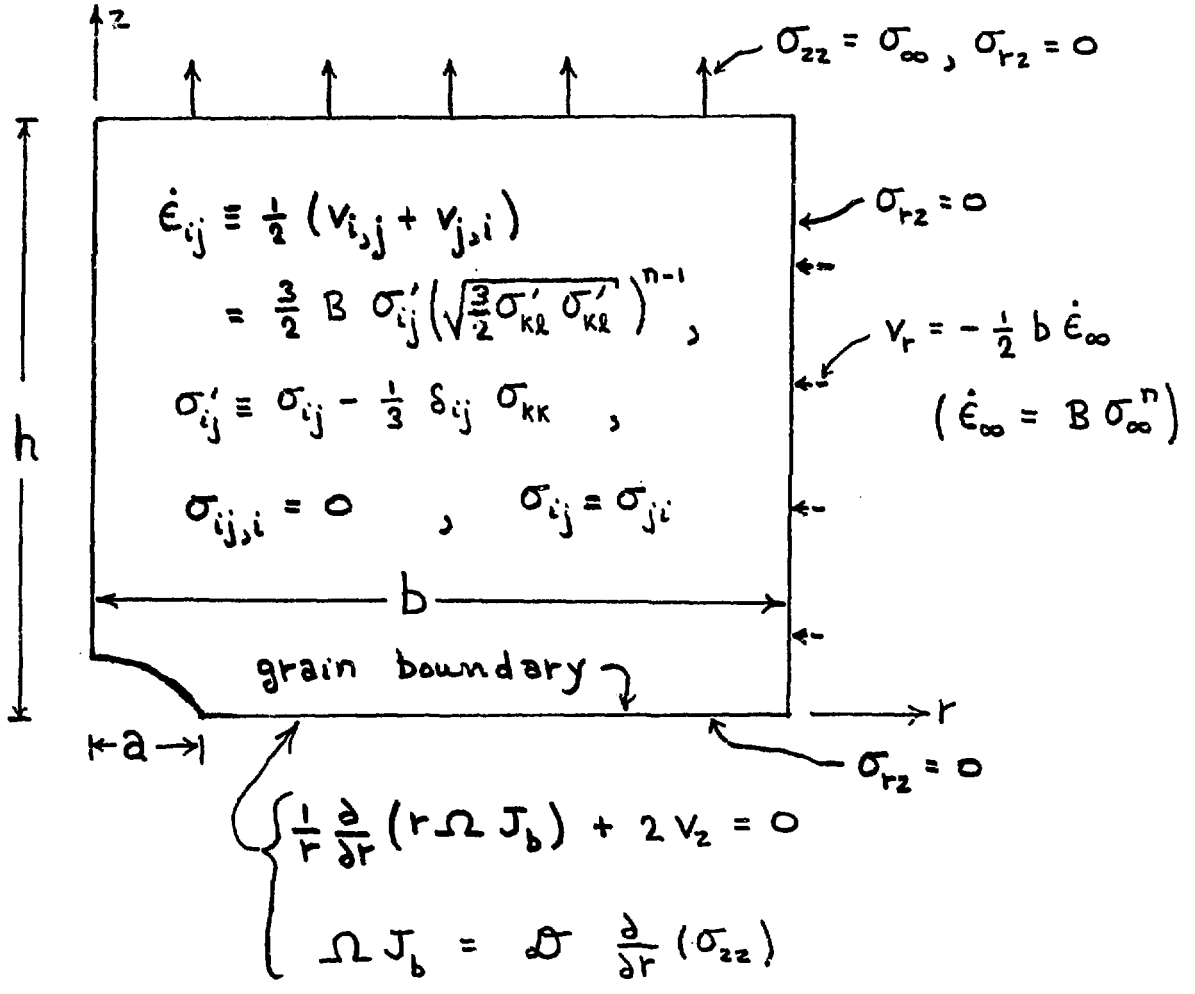

z
$\sigma_{zz} = \sigma_\infty$, $\sigma_{rz} = 0$
$\dot{\epsilon}_{ij} \equiv \frac{1}{2}(v_{i,j} + v_{j,i})$
$= \frac{3}{2} B \sigma'_{ij} \left(\sqrt{\frac{3}{2}\sigma'_{kl}\sigma'_{kl}}\right)^{n-1}$,
$\sigma'_{ij} \equiv \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$,
$\sigma_{ij,i} = 0$, $\sigma_{ij} = \sigma_{ji}$
$\sigma_{rz} = 0$
$v_r = -\frac{1}{2} b \dot{\epsilon}_\infty$
$(\dot{\epsilon}_\infty = B \sigma_\infty^n)$
h
b
grain boundary
r
a
$\sigma_{rz} = 0$
$\frac{1}{r}\frac{\partial}{\partial r}(r\Omega J_b) + 2V_z = 0$
$\Omega J_b = D \frac{\partial}{\partial r}(\sigma_{zz})$

Fig. 3

CONF 790236
UC-25

Time Dependent Fracture of Materials at Elevated Temperature

Proceedings of a Workshop
- — Assessing the State-of-the-Art
- — Recommending Basic Research Relevant to Energy Technologies
- — Held on February 15 - 16, 1979

Editor
Stanley M. Wolf

U.S. Department of Energy
Office of Basic Energy Sciences
Division of Materials Sciences
Washington, D.C. 20545

June 1979

TABLE OF CONTENTS