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Pore pressure effects in inelastic constitutive formulations for fissured rock masses *

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An inelastic constitutive formulation [1,2], developed to relate structural rearrangements of constituent elements of material at the microscale to macroscopic inelastic response, is extended here to the context of fluid-infiltrated porous media. The specific concern is with fissured rock masses and with deduction of the manner in which increments of fluid pore-pressure enter constitutive relations for elastic and inelastic response. The former case has been treated by Biot [3,4]. For the latter case, structural rearrangements related to frictional slippage on fissures and to local cracking at the tips of existing fissures are characterized by internal variables.

At isothermal conditions, the alteration in energy between two adjacent (possibly constrained) equilibrium states has the form, generalized from [1,2] to include pore-pressure effects,

$$d\phi = \sigma_{ij} d\varepsilon_{ij} + p\, dv - \langle f\, d\xi \rangle \quad . \tag{1}$$

Here ϕ is some appropriate Legendre transform of the free energy per unit initial volume, as measured relative to the solid phase, of the two-phase fluid-infiltrated material; ε is the strain of the solid phase, v is the apparent volume fraction [5] of pore space, σ the stress (total), and p the pore pressure. The parameter $d\xi$ measures the extent of some local structural rearrangement (e.g., slippage and/ or further fissuring) and f is the thermodynamic "force" conjugate to that rearrangement; the bracket $\langle \ldots \rangle$ denotes volume average of all such $f\, d\xi$ terms pertaining to all the local structural rearrangements at all sites within a statistically representative macroscopic sample of material. Letting $d^p\varepsilon$ and $d^p v$ be "plastic" increments of strain and pore space associated with structural rearrangements $d\xi$, it follows from (1), in the same manner as for similar derivations in [1,2], that

$$d^p\varepsilon_{ij} = \left\langle \frac{\partial f}{\partial \sigma_{ij}} d\xi \right\rangle \quad , \quad d^p v = \left\langle \frac{\partial f}{\partial p} d\xi \right\rangle \quad , \tag{2}$$

where the partial derivatives are taken at a fixed state of structural arrangement, the f's then being considered functions of σ and p.

Here the plastic increments are defined as the difference between total increments $d\varepsilon, dv$ and those which would result if the response to the corresponding increments $d\sigma, dp$ had been purely elastic. Thus

$$d\varepsilon_{ij} = M_{ijk\ell} d\sigma_{k\ell} + Q_{ij} dp + d^p\varepsilon_{ij}$$

$$dv = R_{ij} d\sigma_{ij} + S\, dp + d^p v$$

where M, Q, R, S are defined as incremental elastic compliances,

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Q and R being symmetric in their indices, M being symmetric in its first and last pairs of indices, and it following also from (1) that

$$M_{ijk\ell} = M_{k\ell ij} \quad , \quad R_{ij} = Q_{ij} \quad .$$

By deducing the form of dependence of thermodynamic forces f on σ and p for representative types of inelastic structural rearrangements, it is possible from (2) to state precise conditions under which

(i) $d^p v = d^p \varepsilon_{kk}$, and

(ii) $d^p \varepsilon_{ij}$ depends on the $d\sigma$'s and dp only through the combination

$$d\sigma_{k\ell} + dp\ \delta_{k\ell} \quad .$$

Statement (i) means that all inelastic dilatant strain occurs via increase of pore space wheras (ii) means that the classical Terzaghi "effective stress" principle is applicable to the inelastic portion of the strain response (although it is not applicable, in general, to the elastic portion [3-6]).

Specifically, to establish conditions for (i) and (ii), we consider fissured, fluid-infiltrated materials in which all pore-spaces are in matter communication with the pore fluid and in which all constituent elements or grains of the solid phase have identical elastic properties as regards their response to (locally) isotropic stress. In such circumstances the response to macroscopic stress and pore pressure increments

$$d\sigma_{ij} = -\ \delta_{ij}\ dq \quad , \quad dp = dq \quad , \tag{3}$$

applied simultaneously, is a local isotropic pressure increase of amount dq at each point of the solid phase (such special σ,p increments are also useful for interpreting elastic response properties, [6,5]). Now, if the forces f conjugate to structural rearrangements are unaffected by this increase,

$$df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial p} dp = \left(-\ \frac{\partial f}{\partial \sigma_{ij}} \delta_{ij} + \frac{\partial f}{\partial p} \right) dq = 0$$

for arbitrary dq , so that the term in parentheses must vanish and hence, by application of (2), statement (i) follows. If the critical conditions for the onset of any set of structural rearrangements $d\xi$ are, likewise, unaffected by isotropic stress increases at each point of the solid phase, then it follows from (2) that statement (ii) holds.

Now consider fissured rock masses under compressive principal stresses and suppose that all inelasticity arises by the processes of (a) frictional slippage at solid-solid contacts on fissure surfaces, and/or (b) further stable cracking from the tips of existing, sharp-tipped fissures. Thermodynamic forces f associated with these processes are readily established by methods of [1,2] (several specific examples are given in [2]). The forces associated with (a) relate to the shear stresses transmitted at contact points, wheras those with (b) are expressed in terms of crack tip stress intensities, via formulae for the crack tip energy release rate. Both types of forces are unaltered by increments of the type (3) so that statement (i) holds. If, further, one adopts with (a) the conventional limiting friction model based on true contact at isolated asperities, comprising a very small fraction of the nominal contact area, and with (b) a model for crack extension based on critical crack tip stress intensities, then increments as in (3) do not affect the conditions for onset of the structural rearrangements and, thus, statement (ii) is valid as well.

References

[1] J. R. Rice, "Inelastic Constitutive Relations for Solids: An Internal Variable Theory and Its Application to Metal Plasticity"; Journal of the Mechanics and Physics of Solids, 19, 1971, pp. 433-455.

[2] J. R. Rice, "Continuum Mechanics and Thermodynamics of Plasticity in Relation to Microscale Deformation Mechanisms"; Chapter 2 of Constitutive Equations in Plasticity (ed. A. S. Argon), M.I.T. Press, 1975, pp. 23-79.

[3] M. A. Biot, "General Theory of Three Dimensional Consolidation", Journal of Applied Physics, 12, pp. 155-164, 1941.

[4] M. A. Biot, "Nonlinear and Semilinear Rheology of Porous Solids", Journal of Geophysical Research, 78, pp. 4924-2937, 1973.

[5] J. R. Rice and M. P. Cleary, "Some Basic Stress Diffusion Solutions for Fluid-Saturated Elastic Porous Media with Compressible Constituents", Reviews of Geophysics and Space Physics, 14, pp. 227-241, 1976.

[6] A. Nur and J. D. Byerlee, "An Exact Effective Stress Law for Elastic Deformation of Rock with Fluids", Journal of Geophysical Research, 76, pp. 6414-6419, 1971.