THE INITIATION AND GROWTH OF SHEAR BANDS

J. R. Rice

Paper published in:

Plasticity and Soil Mechanics (edited by A. C. Palmer), Cambridge University Engineering Department, Cambridge, England, 1973, pp. 263-274. [Proceedings of the Symposium on the Role of Plasticity in Soil Mechanics, Cambridge, 13-15 September, 1973].

THE INITIATION AND GROWTH OF SHEAR BANDS

J. R. Rice†

Palmer and Rice (1973), hereafter designated PR, have proposed a model for the growth of localized shear bands in the progressive failure of overconsolidated clay. Their model entails a gradual decay of strength within the end zone of the shear band, from peak to residual levels, with increasing relative sliding displacement. They derived conditions for propagation of the band by J-Integral methods analogous to Rice's (1968 a,b) treatment of similar cohesive zone models for tensile cracking.

In the present paper the problem of initiation of a shear band is first discussed, giving special attention to the way in which the onset of localized deformation might be viewed as an instability in the constitutive description of homogeneous deformation. Following this, the criteria of PR for growth of an existing band are reviewed and adopted as a starting point for study of some factors that could govern the rate of propagation. These factors were noted in the concluding discussion of PR, as possible sources of the time scale for progressive failure, and include:

(i) Local pore-water suctions and diffusive fluxes induced at the growing end of the band by dilation of the soil as it fails in shear. Here it is envisioned that the induced suction augments the effective compressive stress on the slipping surfaces, thus increasing the shear resistance over what would develop, say, at a lower growth rate for which there would be more time for diffusive equalization of pore pressures.

(ii) Factors which influence the bulk behavior of soil outside the shear band, so as to result in continued relative sliding on the band and thus aid the degradation of shear resistance from peak to residual. This could result from viscoelastic creep of the soil, and also from the time-dependence of deformations due to bulk pore-water diffusion in response to load changes. It could further include Bjerrum's (1967) notion of a tendency for elastic spring-back of overconsolidated clay due to the time-dependent weathering breakdown of soil bonds.

It should be noted that rather similar stabilizing influences of pore-water could occur in dilatant rock masses during faulting. Such effects have been proposed by Frank (1965) and Brace and Martin (1968), and have recently been suggested as a possible source of premonitory warnings of shallow-focus earthquakes (Nur, 1972; Aggarwal et al. 1973) as well as an explanation of aftershock activity (Nur and Booker, 1972).

INITIATION OF SHEAR BANDS

The phenomena of localization of deformation in a band is widely observed in the mechanics of materials and, in general, there are two types of hypothesis that can be developed for it. The first is that some essentially new physical mechanism sets in, abruptly, and degrades the strength of the material. The sudden breaking free and multiplication of pinned dislocations in the Luders yielding of mild steel may be an example

---

of this type.

The alternative hypothesis is that localization can be understood solely on the basis of a smooth continuation of the material's stress-strain relations as they are observed in the pre-failure regime. The notion is that these stress-strain relations may lead to instabilities in mathematical solutions to boundary value problems. In particular, under boundary conditions that are compatible with homogeneous deformation, one may seek a bifurcation point in the solution to the incremental deformation problem, for which the non-uniform mode consists of localization of the continuing deformation in a planar band. Berg (1970) has proposed this type of hypothesis for characterizing the initiation of fracture in ductile metals. He supposes these to contain many small cavities on the microscale which grow with imposed deformation, leading to macroscopic constitutive relations that exhibit dilatational plastic flow and, ultimately, strain softening.

The same analysis could be followed for overconsolidated soils. Indeed, to the extent that a smooth yield locus with plastic normality is applicable, the problem of uniqueness can be posed within Hill's (1958) general variational formulation. But the resulting predictions are very much affected by development of a pointed vertex on the yield locus and by deviations from plastic normality, and no general analysis is yet available. The question of a vertex is important because, within the rigid-plastic approximation, localization can occur only when the bifurcating strain rate field is compatible with the existence of a plane of zero deformation rate. This means that for a material with a smooth yield locus, having ratios of strain rate components completely specified by the current state of stress (by contrast to the situation with a vertex), bifurcation is impossible unless the current state is one for which the intermediate principal strain rate vanishes. When this kinematical condition is met, and normality applies, localization can occur in isotropic materials only when the rate of strain hardening, when phrased in terms of true or Cauchy stress, is non-positive. This was suggested by Berg and can be proven also from Hill's variational formulation (question on graduate plasticity exam, Brown University, January 1972).

A general method for addressing bifurcation conditions, corresponding to localization in a band, is outlined in the remainder of this section. We let σ_{ij} be a state of stress that has been achieved in the uniform deformation of a sample of homogeneous material. The requirement for continuing equilibrium from a uniform stress state is

$$\partial\dot{\sigma}_{ij}/\partial x_i = 0 , \tag{1}$$

where the dot denotes the instantaneous time derivative following a material particle and x_1, x_2, x_3 are cartesian spatial coordinates. Clearly, these equations will be satisfied if the next increment also constitutes uniform deformation, i.e. spatially uniform $\partial v_i/\partial x_j$, where v_i is the instantaneous velocity field.

But we wish to determine circumstances under which equilibrium can be satisfied by a non-uniform field corresponding to variations in the amount of deformation within a planar band. If the coordinate system has been chosen so that the planes x_2 = constant align with the band, the flow field that is sought will be one for which $\partial v_i/\partial x_j$ varies with x_2

within the band, being uniform outside it. It is straightforward to show that the most general compatible flow field of this kind has the form

$$\Delta(\partial v_i/\partial x_j) = g_i(x_2)\delta_{j2} , \qquad (2)$$

where the Δ denotes differences in value between the field within the band and the uniform field outside it, and the functions g_i of x_2 are non-zero only within the band.

Let $\Delta\dot{\sigma}_{ij}$ be the corresponding difference in stress rates. It is clear that this must also satisfy the continuing equilibrium equations (1), and that it is a function of x_2 only, which vanishes outside the band. Hence

$$\Delta\dot{\sigma}_{2j} = 0 \quad , \quad j = 1,2,3 \qquad (3)$$

is necessary and sufficient for maintenance of continued equilibrium at the inception of bifurcation. The rate $\dot{\sigma}_{ij}$ is not invariant to rigid spins, and hence is awkward in constitutive relations. Introducing the Jaumann rate $\overset{\triangledown}{\sigma}_{ij}$, which is that computed by an observer who rotates with the material at an angular velocity formed from the anti-symmetric part of its velocity gradient field (Prager, 1961), (3) becomes

$$\begin{aligned} \Delta\overset{\triangledown}{\sigma}_{21} &= -\frac{1}{2}(\sigma_{22} - \sigma_{11})g_1 + \frac{1}{2}\sigma_{13}g_3 \\ \Delta\overset{\triangledown}{\sigma}_{22} &= \sigma_{21}g_1 + \sigma_{23}g_3 \\ \Delta\overset{\triangledown}{\sigma}_{23} &= -\frac{1}{2}(\sigma_{22} - \sigma_{33})g_3 + \frac{1}{2}\sigma_{31}g_1 \end{aligned} \qquad (4)$$

The analysis can be carried further only with specification of a detailed constitutive law. This will relate $\Delta\overset{\triangledown}{\sigma}_{ij}$ to the g's and, in general, also to the uniform flow field outside the band, in a form for which $\Delta\overset{\triangledown}{\sigma}_{ij}$ vanishes when the g's vanish. If, however, the current state is such that eqs. (4) can be satisfied for some non-zero set of g's , then conditions for flow localization in a band have been met. In the simplest cases these equations involve an eigenvalue problem phrased in terms of parameters of the constitutive rate law, and uncoupled to the field outside the band. This is so for elastic materials, and also for elastic-plastic materials having smooth yield surfaces and plastic strain rate directions determined by the current stress state, provided that the material outside the band continues to load plastically as well. If it unloads, or if the yield surface contains a vertex so that incremental linearity (vs. homogeneity of degree one) does not result for plastic loading, considerable complications result in that the $\Delta\overset{\triangledown}{\sigma}_{ij}$'s will depend on the outside field and vary non-linearly with the g's .

For the rigid-plastic idealization, stress rates are not uniquely expressible in terms of the deformation field and this procedure must be modified. As noted above, a non-uniform flow field is possible only if the strain rates allowable under the current stress state have a surface of zero deformation rate. If this condition is met, the g's can be expressed in terms of the non-uniformity of stress rate and bifurcation conditions are again obtained by requiring that (4) has non-zero solutions.

PR MODEL FOR SHEAR BAND

In any event, once localized deformation has initiated, say by the strain concentration induced by cutting or erosion at the base of a slope, a different constitutive description must be developed for the material of the band. For example, in the PR model the band is treated as a single surface of discontinuity and the description relates the shear stress τ acting on it to the amount of relative sliding displacement δ and prevailing effective compressive stress σ. When viewed from the standpoint of analysis of the surrounding continuum, this constitutive law for the band enters as a coupled boundary condition involving the stress and displacement jump on the surface of discontinuity.

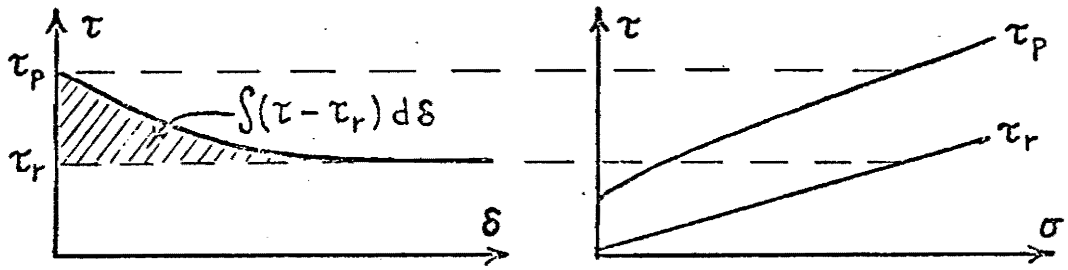

Fig. 1 τ-δ relation for shear band and effect of σ .

Fig. 1 shows a representative τ-δ curve as would be inferred from a shear box or triaxial test on overconsolidated clay, on the assumption that localization has initiated at the peak load with post-peak deformation of the specimen resulting primarily from sliding on a single band. Due to the loss in strength with increasing δ , a shear band, once initiated, can provide a crack-like stress concentration at its tip and hence drive itself on. Indeed, the condition adopted for continued growth of the band is that the concentrated shear stress in material elements at the tip has just been brought up to τ_p , and this can occur for a sufficiently long band under average stresses only slightly above the residual strength τ_r . Further, since τ is related to δ as in fig. 1, the PR model fulfills in a self-consistent manner the suggestion of Bishop (1971) that strength levels along the band should be considered to vary from peak toward residual with increasing distance from the growing end.

An important feature of the model is that it associates a characteristic length with the material. This may, e.g., be identified as the displacement $\bar{\delta}$ defined by

$$(\tau_p - \tau_r)\bar{\delta} = \int(\tau - \tau_r)d\delta \ , \qquad (5)$$

the integral being identified as the crossed area in fig. 1. Typically, from the data of Skempton (1964) and Skempton and Petley (1968) on overconsolidated clays, $\bar{\delta}$ is in the range of 3 to 8 mm. The result of this length scale is that predictions of the model exhibit Griffith-like size effect, with the larger soil masses, having longer shear bands, meeting the propagation criterion at a lower average stress level than would a smaller but geometrically similar mass.

SOME PROPAGATION CRITERIA

Fig. 2 illustrates a long slope, inclined at angle α to the horizontal, into which a step of height h has been cut. A shear band of length ℓ has progressed upward from the base of the cut, paralleling the ground surface. This was analyzed approximately in PR through use of the J-Integral, with the assumption that h and ω are small in comparison to ℓ. Here ω is the size of the end zone near the tip, beyond which the shear stress τ is essentially equal to τ_r.

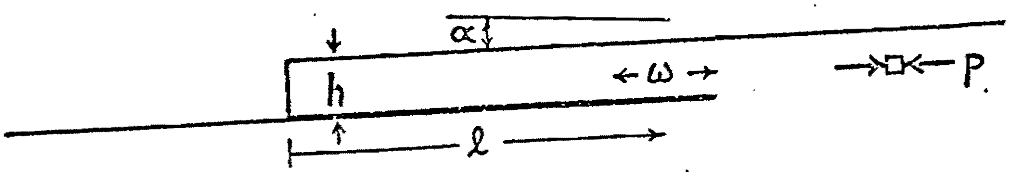

Fig. 2 Shear band emanating from step in long slope.

The principal approximation is in assuming that the overhanging layer can be treated as a one dimensional element, whose downslope extensional strain $\bar{\varepsilon}$ at any point along its length is given in terms of the average $\bar{\sigma}$ of the tensile stress acting through the depth h at that point: $\bar{\varepsilon} = \bar{\varepsilon}(\bar{\sigma})$. The strain is measured from zero when $\bar{\sigma} = -p$, where p is the average compressive stress acting over the depth h at points uphill from the perturbation caused by the shear band. Hence, within the one-dimensional approximation, $\bar{\sigma} = -p$ just before the end zone reaches a material point, and $\bar{\sigma} = (\tau_g - \tau_r)\ell/h$ just after the end zone passes by, where $\tau_g = \bar{\rho}gh \sin\alpha$ is the downslope shear stress that would be exerted at depth h by gravitational loadings, in the absence of the step or shear band.

The resulting criterion which must be met for the band to be just able to propagate is, from PR,

$$h \int_{-p}^{(\tau_g - \tau_r)\ell/h} \bar{\varepsilon}(\bar{\sigma})\,d\bar{\sigma} = \int (\tau - \tau_r)\,d\delta \quad , \quad \text{or} \tag{6}$$

$$\frac{\tau_g + p\,h/\ell - \tau_r}{\tau_p - \tau_r} = \left(\frac{2E'}{\tau_p - \tau_r} \frac{\bar{\delta} h}{\ell^2} \right)^{1/2} , \tag{7}$$

where in the latter form (5) is used and the stress-strain relation for the layer in its transition from $-p$ to $(\tau_g - \tau_r)\ell/h$ is linearized to $\bar{\sigma} = -p + E'\bar{\varepsilon}$. In (7) it may be noted that $\tau_g + ph/\ell$ is the average shear stress which could be considered to act on the band, due to the gravitational loading and to the side force of the initial lateral pressure p. Thus we see the size effect: for small $\bar{\delta}$, or for large overall dimensions of the slope, the propagation criterion can be met at an average stress that is well below τ_p and only slightly above τ_r. Alternatively, if $\bar{\delta}$ and the physical dimensions are such that the right side of (7) is near to or greater than unity, the small end zone approximation in the analysis is untenable and propagation occurs under an average stress near τ_p. We may note also that even if the slope angle is such that $\tau_g = \tau_r$, a sufficiently large initial lateral stress p will drive the band indefinitely.

The energy balance interpretation of the PR propagation criterion, discussed in general terms in their Appendix, is made evident after multiplying by $d\ell$ in (6), integrating by parts on the left and rearranging it to read

$$(\tau_g \ell)(\bar{\varepsilon}_\ell d\ell) = h d\ell \int_0^{\bar{\varepsilon}_\ell} \bar{\sigma}(\bar{\varepsilon}) d\bar{\varepsilon} + (\tau_r \ell)(\bar{\varepsilon}_\ell d\ell) + d\ell \int (\tau - \tau_r) d\delta \ . \tag{8}$$

Here $\bar{\varepsilon}_\ell$ is the strain at the point which has just been passed by the end zone, and at which $\bar{\sigma} = (\tau_g - \tau_r)\ell/h$. Thus if the band advances by $d\ell$ the overhanging layer beyond the end zone moves downslope uniformly by $\bar{\varepsilon}_\ell d\ell$. From the definition of τ_g, $\tau_g \ell$ is the net downslope gravitational force on the overhang and hence the left side of (8) is the work input during the movement. This should balance against the stress work of deforming the material plus the frictional dissipation on the shear band and, indeed, the right side of (8) has exactly this interpretation: The first term is the stress work in bringing the element $h d\ell$ of overhang to the strain $\bar{\varepsilon}_\ell$, whereas the second term represents frictional dissipation against the residual part of the shear strength and the final term, involving the shaded area of fig. 1, represents dissipation against strengths in excess of the residual level. This interpretation makes it clear that the propagation criterion as given by (6,7) is, within the one-dimensional approximation, valid even if the overhang deforms inelastically, so long as the relation $\bar{\varepsilon} = \bar{\varepsilon}(\bar{\sigma})$ employed in the criterion represents the actual strain evolution as the stress increases from $-p$ to $(\tau_g - \tau_r)\ell/h$ over the end zone size ω. Of course, the one-dimensional approximation itself is more justified in the elastic case because then the stress work in non-uniform deformations near the tip is fully recovered.

A general analysis was also given in PR for the case in which the surrounding continuum is regarded as linear elastic and the end zone ω is taken to be small in comparison to the shear band length and other dimensions of the soil mass. In that case the propagation criterion is phrased in terms of the shear mode stress intensity factor K (see, e.g., Rice, 1968 a) as computed from the singularity of the elastic stress field solution that results when a vanishingly small end zone is assumed, so that τ is taken to be τ_r everywhere on the band. The result for an isotropic material is

$$K^2/E' = \int (\tau - \tau_r) d\delta \ , \tag{9}$$

where E', as earlier, is the plane strain tension modulus and is related to the shear modulus G and contraction ratio ν by $E' = 2G/(1-\nu)$.

The elastic singularity characterized by K is actually considered to be annulled, in the manner of Barenblatt (1962), by that of opposite sign induced by resisting stresses in excess of τ_r within the end zone. Because of the assumed smallness of the end zone, this can be computed from the formula for surface loads on a semi-infinite shear crack in an infinite body, and the result is

$$K = (2/\pi)^{1/2} \int_0^\infty R^{-1/2} [\tau(R) - \tau_r] dR \tag{10}$$

where $\tau(R)$ is the stress at distance R from the tip of the band.

One may also compute the relative sliding displacement δ near the tip of the band from the same semi-infinite crack model. The result for δ at a distance S from the tip of the band is, when (10) is used to eliminate an explicit dependence on K,

$$\delta(S) = (4/\pi E')\int_0^\infty [\tau(R)-\tau_r]\{2(S/R)^{1/2} - \log[1+(S/R)^{1/2}] + \log|1-(S/R)^{1/2}|\}dR \,, \tag{11}$$

as may, e.g., be developed from methods of crack elasticity given by Rice (1968 a). By assuming a linear variation of τ with R, from τ_p to τ_r over the distance ω, and requiring that the result of (10) satisfy the propagation criterion (9), the end zone size was estimated in PR as

$$\omega = \frac{9\pi}{16(1-\nu)} \frac{G}{\tau_p-\tau_r} \bar{\delta} \,. \tag{12}$$

With G taken as the loading modulus for London clays studied by Wroth, this gave $\omega \approx 250\,\bar{\delta}$ when τ_r is half τ_p, suggesting end zone sizes from 3/4 to 2 m for the range of $\bar{\delta}$ noted earlier. Larger values result if G is taken as the unloading modulus.

The discussion of time effects is most easily organized around (6). The rate factor listed as (i) on the opening page amounts to altering the right side of (6) (i.e. the crossed area of fig. 1) because τ is elevated locally by the induced suctions. Thus this term, representing the energy which must be supplied to the band for dissipation against strengths in excess of τ_r, will be an increasing function of the growth rate. Those factors listed as (ii) affect only the left side of (6). In particular, viscoelastic and bulk diffusion effects alter the strain $\bar{\varepsilon}_\ell$ produced by the given stress variation, and hence the energy that can be made available to supply the required end zone dissipation. These effects are such that the energy that can be supplied decreases with growth rate. Bjerrum's notion of weathering would increase both the initial stress p and the springiness with which it is released, and thus increase the energy supply. To the extent that such a process will actually take place, it is possible that it could be initiated or very much augmented by extensional straining of ground near the growing tip of the band, and hence contribute to determining the rate of growth in a manner similar to that of viscoelastic effects.

INDUCED SUCTIONS IN A DILATING END ZONE

The analysis of shear failure at the end of a band in saturated soil, with local dilation and consolidation effects, poses a formidable problem, especially because of the coupling between the induced effective compressive stress and the resistance to continued sliding in the band. The approach taken here is very much simplified.

Let H denote the total volume of water that has been drawn to a unit area of the shear band by dilation within the end region. For convenience, the content is taken to rise linearly with distance over the end region size ω, as in fig. 3. Thus when the band is growing at a steady speed V, fluid is being drawn from soil on both sides of the band at an apparent velocity $(H/2)V/\omega$.

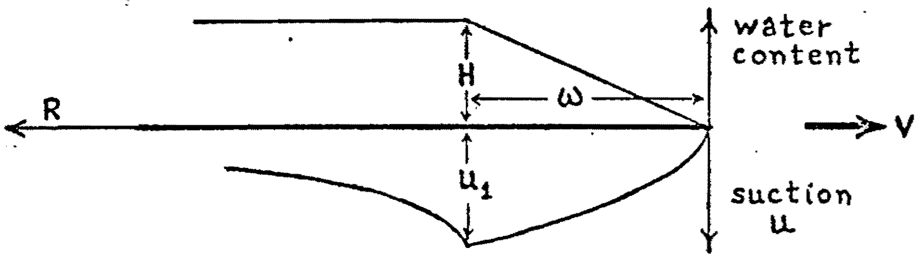

Fig. 3 Water content from dilation and induced suction.

If fluid diffusion in the soil in directions parallel to the band is ignored and if the total normal stress on the band is assumed to remain constant, the fluid suctions u induced by this withdrawal can be calculated approximately from the one-dimensional consolidation/swelling equation. Under well known assumptions, this is

$$\partial^2 u/\partial y^2 = (M\gamma/k)\partial u/\partial t \quad , \tag{13}$$

where y is measured perpendicular to the band, M is compressibility, γ the weight density of the fluid, and k the permeability or apparent velocity per unit gradient in pressure head. From standard diffusion solutions, the suction on the plane of withdrawal that is required to maintain the above constant rate, starting at $t = 0$, is

$$u = \frac{(H/2)V/\omega}{k/\gamma}\left(\frac{4\ k\ t}{\pi\ \gamma\ M}\right)^{1/2} \tag{14}$$

Identifying $t = R/V$ from fig. 3, and noting that the cessation of withdrawal at $R = \omega$ is describable by superposition of the negative of the above solution with a shifted time origin, the induced suction distribution along the shear band is

$$u = u_1\ (R/\omega)^{1/2} \text{ for } 0 < R < \omega \ ; \quad u = u_1\ [(R/\omega)^{1/2} - (R/\omega - 1)^{1/2}] \text{ for } R > \omega \ , \tag{15}$$

as shown in fig. 3, where the maximum suction is

$$u_1 = H\ (\gamma V/\pi k M\omega)^{1/2} \tag{16}$$

For example, taking $H = 2$ mm, $k = 10^{-8}$ m/sec , $M = .02$/bar as may be appropriate for elastic swelling of stiff clay, $\omega = 1.5$ m as representative of the above estimates, and γ for water,

$$u_1 \approx 20 \text{ bars } (V/\text{m/sec})^{1/2} \approx .07 \text{ bar } (V/\text{m/day})^{1/2} \tag{17}$$

Thus speeds in the neighborhood of 1 m/day or larger might induce suctions that could have discernible effects on shear resistance.

To estimate the resulting effect in the propagation criterion for the band, the linear elastic model with a small end zone is considered. We simplify the $\tau - \delta$ relation of fig. 1 to a step form by assuming that, in the absence of induced suctions, a constant τ develops for δ less than a certain critical value and that τ drops abruptly to τ_r beyond this value:

$$\tau = \tau_r + \frac{2}{3}(\tau_p - \tau_r) \text{ for } \delta < \frac{3}{2}\bar{\delta} \quad ; \quad \tau = \tau_r \text{ for } \delta > \frac{3}{2}\bar{\delta} \ . \tag{18}$$

This choice of stress level and critical δ preserves the value of the shaded area in fig. 1, which is the most important parameter in the propagation criterion for the small end zone case; the factor 2/3 of $(\tau_p-\tau_r)$ is chosen as this ensures that the end zone size ω computed for this $\tau-\delta$ relation agrees exactly with (12).

The induced suction u increases the effective compressive stress on the band by u, and this increases τ as shown on the right in fig. 1. The amount of increase is μu, where μ is the coefficient of friction ($= \tan \phi$). We take a constant value μ_p for the near-peak range $\delta < 3\bar{\delta}/2$, and another constant value μ_r for the residual range $\delta > 3\bar{\delta}/2$. Thus, adding the terms μu to τ as given by (18), we have τ as a function of u and δ. The unknown of the problem is the distance ω from the tip of the band at which δ reaches the critical value, but we can write the stresses acting in terms of ω from (18) and (15):

$$\begin{aligned} \tau-\tau_r &= \frac{2}{3}(\tau_p-\tau_r) + \mu_p u_1 (R/\omega)^{1/2} \quad \text{for} \quad 0 < R < \omega \\ \tau-\tau_r &= \mu_r u_1 [(R/\omega)^{1/2} - (R/\omega-1)^{1/2}] \quad \text{for} \quad R > \omega . \end{aligned} \qquad (19)$$

Thus, by seeing that the δ as calculated from this stress distribution by (11) agrees with the critical value at a distance $S=\omega$ from the tip, we obtain the following equation to determine ω:

$$\frac{3}{2}\bar{\delta} = \frac{2(1-\nu)}{\pi G} \left\{\frac{4}{3}(\tau_p-\tau_r)\omega + \mu_p u_1 \omega (1-\log 2) - \mu_r u_1 \omega \log 2\right\} \qquad (20)$$

Recognizing that u_1 is itself dependent on ω, this can be solved to give

$$\omega = \omega^o [(1+\beta^2)^{1/2}-\beta]^2, \quad \text{where} \quad \beta = \frac{3}{8}\frac{\mu_p(1-\log 2)-\mu_r \log 2}{\tau_p-\tau_r} u_1^o, \qquad (21)$$

where ω^o is the value of ω given by (12), when there are no suction effects, and where u_1^o is the value of the maximum suction that would be computed from (16) when ω^o is inserted for ω. Since β vanishes when $\mu_r = .44\,\mu_p$ and since β is still very small for typical μ's even when the velocity is so great that the augmented shear resistance $\mu_p u_1^o$ has become comparable in value to τ_p, we can conclude that the size ω of the end zone will be virtually unaffected by the induced suction.

What is affected, however, is the stress intensity factor K necessary to drive the band. This is computed from (10) in terms of the shear stresses of (19). The integral is formally divergent, in a logarithmic fashion, and thus we cut off the assumed suction distribution at some distance ℓ from the tip, where ℓ can be identified as the length of the band or, better, the increase in length that has taken place while the band has been growing at velocities comparable to the current V. Thus one obtains

$$K = \left(\frac{2\omega}{\pi}\right)^{1/2}\left[\frac{4}{3}(\tau_p-\tau_r) + \mu_p u_1 + \mu_r u_1 \left\{\frac{\ell}{\omega} - 1 - \frac{\ell}{\omega}\left(1-\frac{\omega}{\ell}\right)^{1/2} + \log\left[\left(\frac{\ell}{\omega}\right)^{1/2} + \left(\frac{\ell}{\omega}-1\right)^{1/2}\right]\right\}\right] \qquad (22)$$

The terms in $\{\ldots\}$ approach $\log(4\ell/e\omega)^{1/2}$ when ℓ/ω is large, and differ from this by only a few percent even when $\ell = 2\omega$; hence the latter form will be used subsequently.

Of more fundamental interest is the augmentation of the resistance term $\int(\tau-\tau_r)d\delta$ which arises from the local suctions, since this enters the general propagation criterion in PR. The term can be computed from the above K through (9) and, given the earlier remarks, we can use (12) for ω to simplify the expression. The result is

$$\int(\tau-\tau_r)d\delta = (\tau_p-\tau_r)\bar{\delta}\left\{1 + \frac{3\ \mu_p u_1}{4\ (\tau_p-\tau_r)}\left[1 + \frac{\mu_r}{\mu_p}\log\left(\frac{4\ell}{e\omega}\right)^{1/2}\right]\right\}^2 \qquad (23)$$

The dependence on ℓ is not very strong: e.g. if $\mu_r = (2/3)\mu_p$, which is a typical ratio, the values of $[\ldots]$ for $\ell/\omega = 4$, 20, 100, and 500 are, respectively, 1.6, 2.1, 2.7, and 3.2 . Thus we see that the energy which must be supplied to the end region, for dissipation against strength levels in excess of τ_r , could be approximately doubled if there were sufficient suction u_1 induced to make the augmented shear stress $\mu_p u_1$ equal to about 20 to 25% of $\tau_p-\tau_r$. For example, using the numerical values leading to (17) and taking $\mu_p = \tan(25°)$, $\tau_p-\tau_r = 0.3$ bar , this doubling of the dissipation would occur at a growth rate of approximately 4 m/day.

There is, of course, considerable latitude in choosing numerical values in these formulae. But the conclusion would seem to be that dilationally induced suctions could govern the rate of progressive failure of a slope only during the terminal stages of shear band growth, involving a time scale on the order of one to several days.

EFFECTS OF BULK TIME DEPENDENCE

Effects on longer time scales are, instead, probably explainable in terms of time-dependence in bulk material behavior, although there is also the possibility of creep-like effects in the τ-δ relation for the band itself. With reference to the slope of fig. 2 and the corresponding propagation criterion (6,7), it is evident that if the band grows with speed V , then the constitutive properties (i.e., E' in the linear case) used in the criterion should be those appropriate to a time scale of order ω/V , over which the main stress alterations take place.

A similar choice of material properties, based on ω/V , is suggested by solutions to cohesive zone models for steady speed tensile crack growth in isotropic, linear viscoelastic materials (e.g., Barenblatt et al. 1970; Wnuk and Knauss, 1970). When these are adapted to the shear mode, in the context of a small end zone model paralleling the elastic results (9-11), and when the τ-δ relation is simplified to that in (18), the propagation criterion has the same form as (9), namely

$$K^2/E'(\omega/V) = (\tau_p-\tau_r)\bar{\delta} \quad , \qquad (24)$$

where now $1/E'(\omega/V)$ denotes the creep compliance function for plane-strain tension, evaluated at a time which is a materially-dependent fraction of ω/V . This time is $\omega/3V$ for the Maxwell model. The K which appears is the stress intensity factor of the singular viscoelastic stress field, computed as if there were no end zone; it corresponds with that of the elastic solution in certain traction boundary value problems. The distance ω at which the critical δ is reached is now no longer a material constant, but it is related to K by (10) which remains valid in the viscoelastic case. Thus

$$\omega \approx (9\pi/32)\, K^2/(\tau_p-\tau_r)^2 \qquad (25)$$

and this together with (24) enables the calculation of the K necessary to drive the band at any given velocity V.

Bishop (1968) has summarized some creep data of Lovenbury on London clay, suggesting strain increases of approximately 7% and 25% over the 3 day value in 30 days and 300 days, respectively. These same percentages may be taken as the approximate amounts by which E' should be regarded to have decreased from its 3 day value for end zone processes on the two longer time scales. Hence (7) would predict that the required excess of the average shear stress over τ necessary to drive the band is about 3% and 12% less, respectively, for the two longer time scales than for the 3 day scale. Given that end zone sizes of order 1 m are expected, and equating the time scales to ω/V, this data suggests that creep effects would require a 10-15% increase in average shear stress excess to bring V from speeds of order 1 m/year to those of order 1 m/day.

Finally, to briefly consider effects of bulk diffusion, suppose that the slope overhang of fig. 2 is regarded as a porous, linear elastic material with shear modulus G. Then the plane-strain tension modulus E' would decrease from $4G$ under completely undrained, short-time conditions to $2G/(1-\nu)$ for completely drained, long-time conditions, where ν is the drained contraction ratio. If $\nu = .2$ this would be a 37% drop and would account in (7) for a 21% drop in the excess of the average shear stress over τ_r that is needed to propagate the band. The amount of this drop that could actually be realized in propagation at speed V would, of course, depend approximately on the dimensionless combination of diffusivity multiplied by the characteristic straining time ω/V, divided by the square of the shorter of the two diffusion path-lengths, ω and h.

ACKNOWLEDGMENT: This work was supported by the ARPA/NSF Materials Research Laboratory at Brown University. I am grateful to M. Cleary and A. C. Palmer for several helpful discussions.

REFERENCES

Aggarwal, Y. P., Sykes, L. R., Armbuster, J. & Sbar, M. L. (1973). Premonitory changes in seismic velocities and prediction of earthquakes. Nature 241, 101-104.

Barenblatt, G. I. (1962). Mathematical theory of equilibrium cracks in brittle fracture. Advances in Applied Mech., Vol. 7. pp. 55-129. Academic Press.

Barenblatt, G. I., Entov, V. M. & Salganik, R. L. (1970). Some problems of the kinetics of crack propagation. Inelastic Behavior of Solids. pp. 559-584. McGraw-Hill.

Berg, C. A. (1970). Plastic dilation and void interaction. Inelastic Behavior of Solids. pp. 171-209. McGraw-Hill.

Bishop, A. W. (1968). Discussion. Proc. Geotechnical Conf., Oslo, 1967; Vol.2. pp. 153-154. Norwegian Geotechnical Institute.

Bishop, A. W. (1971). The influence of progressive failure on the choice of the method of stability analysis. Geotechnique 21, 168-172.

Bjerrum, L. (1967). Progressive failure in slopes of overconsolidated plastic clay and clay shales. Trans. Am. Soc. Civ. Engrs. SM 93, 3-49.

Brace, W. F. & Martin, R. J. (1968). A test of the law of effective stress for crystalline rocks of low porosity. Int. J. Rock Mech. Min. Sci. 5, 415-426.

Frank, F. C. (1965). On dilatancy in relation to seismic sources. Rev. Geophys. 3, 485-503.

Hill, R. (1958). A general theory of uniqueness and stability in elastic-plastic solids. J. Mech. Phys. Solids 6, 236-249.

Nur, A. (1972). Dilatancy, pore fluids, and premonitory variations of t_s/t_p travel times. Bull. Seism. Soc. Am. 62, 1217-1222.

Nur, A. & Booker, J. R. (1972). Aftershocks caused by pore fluid flow. Science 175, 885-887.

Palmer, A. C. & Rice, J. R. (1973). The growth of slip surfaces in the progressive failure of overconsolidated clay. Proc. Roy. Soc. Lond. A 332, 527-548.

Prager, W. (1961). Introduction to the Mechanics of Continua. Ginn and Co.

Rice, J. R. (1968 a). Mathematical analysis in the mechanics of fracture. Fracture: An Advanced Treatise, Vol.2. pp. 191-311. Academic Press.

Rice, J. R. (1968 b). A path independent integral and the approximate analysis of strain concentration by nothces and cracks. J. Appl. Mech. 35, 379-386.

Skempton, A. W. (1964). Long term stability of clay slopes. Geotechnique 14, 77-101.

Skempton, A. W. & Petley, D. J. (1968). The strength along structural discontinuities in stiff clays. Proc. Geotechnical Conf., Oslo, 1967; Vol.2. pp. 29-46. Norwegian Geotechnical Institute.

Wnuk, M. P. & Knauss, W. G. (1970). Delayed fracture in viscoelastic-plastic solids. Int. J. Solids Structures 6, 995-1009.