Pageoph, Vol. 115 (1977), Birkhäuser Verlag, Basel

Electromechanical Effects Associated with Earthquakes

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Summary – A theoretical model of an electroelastic continuum has been applied in this paper to the problem of electrodynamic phenomena (piezoelectricity, electrostriction, etc.) associated with earthquakes. In such a model the coupling between electric and mechanical fields expresses itself by a change of scale of mechanical effects along the electric field, as well as by the additional electric charge created by the earthquake source.

The electrokinetic phenomena associated with earthquakes and caused by the diffusion of fluids into the dilatant region have been considered using the theory of porous media with interstitial fluid flow. General relations describing electrokinetic effects caused by the deformation processes in an earthquake source have been obtained.

Key words: Electromechanical effects in earthquakes; Earthquake focus; Electromechanical effects.

1. Introduction

Recent work on the earthquake physics, and especially on the earthquake mechanism, revealed a number of different physical effects closely connected with the earthquakes; changes of seismic velocities, electric resistivity, electrotelluric potentials, electric potential of air, rate of flow of water, tilts, magnetic anomalies, radon concentration in hydrothermal wells, etc. All these effects including the electromechanical ones have been extensively studied, mainly for the purpose of earthquake prediction. The problem has been approached mainly from two directions: field measurements of different electric effects associated with earthquakes (e.g. RIKITAKE and YAMAZAKI, 1969; MYACHKIN *et al.*, 1972; MYACHKIN and ZUBKOV, 1973; MAZZELLA and MORRISON, 1974; YAMAZAKI, 1974; SOBOLEV, 1975; SOBOLEV *et al.*, 1975; YAMAZAKI, 1975), and laboratory studies to explain physical basis of the observed phenomena (e.g. YAMAZAKI, 1965, 1966, 1967, 1968; PARKHOMENKO, 1967; BRACE, 1968; BRACE and ORANGE, 1968; MITCHELL and BRACE, 1973; BRACE, 1975; SOBOLEV, 1975). However, there still is no adequate theoretical description of electromechanical effects associated with earthquakes. This paper attempts to describe such a theory. The theory should help proper interpretation of the electromechanical phenomena associated with earthquakes.

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2. *Electroelastic field of earthquake source*

The present attempt of theoretical investigation of electroelastic effects for a source defined by a set of point forces, in particular for a double dipole with moment, is confined to the stationary processes only. The medium is treated as purely isotropic. Since the following numerical example is based on approximate estimates of the material constants, the value of the present analysis is reduced to the general investigation of the character of analyzed phenomenon, its dependence on the energy of seismic source and rough estimation of electric effects.

The starting point of the present work is a model of an electrically polarizable, finitely deformable elastic continuum, presented by TIERSTEN (1971). The model consists of an electric charge continuum coupled to a mechanical (lattice) continuum.

The nonlinear equations for the electroelastic continuum have been derived by Tiersten by the systematic use of conservation laws to both coupled continua, using the infinitesimal displacement field $\eta(x, t)$, to describe the displacement of electronic continuum with respect to the lattice continuum.

The equations derived by Tiersten are as follows:

$$\left.\begin{aligned} &\tau_{ij,i} + \rho\pi_i E_{j,i} = \rho\ddot{u}_j \qquad \tau_{ij}^A = \frac{\rho}{2}[E_i\pi_j - \pi_i E_j] \\ &\rho\mathcal{T} = \rho_0 \qquad D_{i,i} = 0 \qquad E_i = -\varphi_{,i} \end{aligned}\right\} \tag{1}$$

where τ_{ij}^A = antisymmetric part of the stress tensor τ_{ij};
π_i = polarization vector per unit mass $\pi_i = P_i/\rho$, where P_i is the polarization and ρ the density;
E_i = electric field (the time variations are assumed quasi-static);
ρ_0 = mass density in the reference configuration;
φ = electric potential.

Here $\mathcal{T} = \det x_{i,L}$ and the upper case letters denote material coordinates X_L (reference configuration), while the lower case letters denote the space coordinates x_i describing the deformation

$$\left(f_{,L} = \frac{\partial f}{\partial X_L}, \qquad g_{,i} = \frac{\partial g}{\partial X_i}\right).$$

The above equations should be completed by the constitutive equations as well as the heat conducting constitutive equation, which satisfies the entropy inequality

$$-q_i\theta_{,i} \geq 0,$$

where q_i describes the heat flux and θ is the positive absolute temperature.

The constitutive equations could be determined by a proper choice of a thermodynamical function ψ. In particular, for a simple medium it is very convenient to use the generalized function of a free energy:

$$\psi = \varepsilon - \eta\varepsilon - E_i \pi_i,$$

where ε is the inner energy and η is entropy.

The constitutive relations give the following equations:

$$\tau_{ij} = \rho x_{i,M} \frac{\partial \psi}{\partial x_{j,M}}, \qquad \pi_i = -\frac{\partial \psi}{\partial E_i}, \qquad \eta = -\frac{\partial \psi}{\partial \theta}.$$

As has been shown by Tiersten, in general the function ψ could be an arbitrary function depending on the deformation tensor e_{LK}, the vector $w_L = x_{n,L} E_n$ and the temperature θ, thus

$$\psi = \psi(e_{LK}, w_L, \theta)$$

and the constitutive equations would be then of the following form:

$$\tau_{ij} = \rho x_{i,M} \frac{\partial \psi}{\partial e_{MN}} x_{j,N} + \rho x_{i,M} \frac{\partial \psi}{\partial w_M} E_j,$$

$$\pi_i = -x_{i,L} \frac{\partial \psi}{\partial w_L},$$

$$\eta = -\frac{\partial \psi}{\partial \theta}.$$

The heat flux vector q_i will be, in the general case, given by a following relation

$$q_i = x_{i,n} L_n,$$

where

$$L_n = L_n(\theta_{,M}, e_{LM}, w_L, \theta).$$

For linear heat conduction L_k assumes the following form

$$L_k = K_{kN} \theta_{,N}$$

A reasonable polynomial approximation for the thermodynamical function ψ for thermo-electro-elastic bodies is of the form (TIERSTEN, 1971):

$$\psi = \frac{1}{2\rho_0} c_{KLMN} e_{KL} e_{MN} - \frac{1}{2\rho_0} \kappa_{KL} w_K w_L - \tfrac{1}{2} c\theta^2 - \frac{1}{\rho_0} \alpha_{KLM} w_K e_{LM}$$
$$- \frac{1}{2\rho_0} \beta_{KLMN} w_K w_L e_{MN} + \frac{1}{\rho_0} \gamma_{KL} e_{KL} + \frac{1}{\rho_0} \lambda_K w_K \theta + \cdots$$

where the material coefficients c_{KLMN}, κ_{KL}, c, α_{KLM}, β_{KLMN}, γ_{KL} and λ_K are called the elastic, electric susceptibility, thermal, piezoelectric, electrostrictive, thermo-elastic and pyroelectric constants respectively. They can be functions of the absolute temperature θ, and in some cases even functions of the electric field through w_K.

We will confine ourselves now to the linear theory only, we will also neglect the thermal effects. For the case of perfect elasticity the free energy will be of the form ($\kappa_{kl} = \kappa\delta_{kl}$, $\beta_{klmn} = \beta\delta_{kl}\delta_{mn}$)

$$\psi = \frac{1}{2\rho}(\lambda e_{nn}e_{ss} + 2\mu e_{kl}e_{kl}) - \frac{\kappa}{2\rho}E_kE_k - \frac{1}{\rho}\alpha_{kls}E_ke_{ls} - \frac{\beta}{2\rho}E_kE_ke_{ss}. \tag{2}$$

The assumption of the elastic isotropy practically eliminates from our considerations all piezoelectric bodies, since piezoelectric effects are caused by the elastic anisotropy. Consequently, we shall neglect the term accounting for the piezoelectric effect, i.e. α_{kls}, and leave terms connected with electrostrictive effects only, independent of the direction of the electric field E_i. However, it seems to be interesting to preserve all terms of equation (2), including piezoelectric effects, when discussing this equation and its interpretation by the use of approximate solutions. Thus we will follow this scheme, neglecting the piezoelectric effects only during the further exact computations.

For the linear theory we shall obtain:

$$\left.\begin{aligned}
\pi_i &= \frac{P_i}{\rho} = -\frac{\partial\psi}{\partial E_i} = \frac{\kappa}{\rho}E_i + \frac{\alpha_{ils}}{\rho}e_{ls} + \frac{\beta}{\rho}E_ie_{ss},\\
\tau_{ij} &= \rho\frac{\partial\psi}{\partial e_{ij}} + \rho\frac{\partial\psi}{\partial E_i}E_j = \lambda e_{ss}\delta_{ij} + 2\mu e_{ij} - \kappa E_iE_j\\
&\quad - \alpha_{i(ls)}E_je_{ls} - \beta E_iE_je_{ss} - \alpha_{k(ij)}E_k - \frac{\beta}{2}E_nE_n\delta_{ij},\\
\tau^A_{ij} &= \tau_{[ij]} = -\tfrac{1}{2}e_{ls}[\alpha_{ils}E_j - \alpha_{jls}E_i],\\
\alpha_{ils} &= \alpha_{i(ls)}.
\end{aligned}\right\} \tag{3}$$

Let us assume now, that the comparatively small field connected with the stresses in an earthquake focus, disturbs the constant regional electric field E_i. Thus we shall put $E_i + \varepsilon_i$ instead of E_i, and we will confine ourselves to the linear expressions only. The law of momentum conservation can therefore be expressed

$$\tau_{ij,i} + P_i\varepsilon_{j,i} = \rho u_j.$$

Introducing the earlier expressions for the stress tensor we shall obtain

$$\tau^0_{ij,i} - E_j(\kappa\varepsilon_{i,i} + \alpha_{ils}e_{ls,i} + \beta E_ie_{ss,i}) - \beta E_k\varepsilon_{k,j} - 2\varepsilon_{k,i}\alpha_{kij} + \rho f_j = \rho\ddot{u}_j, \tag{4}$$

where τ^0_{ij} is the purely mechanical part of the stress tensor and accounts for the elastic medium.

Analogously, introducing

$$D_i = (E_i + \varepsilon_i) + 4\pi P_i,$$

we will obtain the following form for the equation $D_{i,i} = 0$

$$-\varphi_{,ii} + 4\pi(\alpha_{ls,i} + \beta E_i e_{ss,i}) = 0, \tag{5}$$

where $\varepsilon_i = -\varphi_{,i}$.

Equations (4) and (5) are the starting point for the further discussion of the relation between the field ε_i and the seismic source.

The above equations could be solved approximately in the following way: we assume that the first-step approximation of the deformation field is given by the Love's solution for the point sources τ_{ij}^1, u_i^1:

$$\tau_{ij,i}^1 + \rho f_j^{\,1} = \rho \ddot{u}_j^1.$$

The conjugated electric field ε_i^1 can be obtained from the Poisson's equation

$$\varphi_{,ii}^1 = 4\pi(\alpha_{ils} e_{ls,i}^1 + \beta E_i e_{ss,i}^1). \tag{6}$$

It should be noted that the right side of equation (6) is given by the known deformation field for the point solutions given by Love, e_{is}^1.

The second-step correction in the stress field, τ_{ij}^2, could be obtained from the equation

$$\tau_{ij,i}^2 + \rho f_j^{\,2} = \rho \ddot{u}_j^2, \tag{7}$$

where $\rho f_j^{\,2}$ is the additional force, acting in an earthquake focus and determined by the known values e_{ij}^1 and ε_i^1:

$$\rho f_j^{\,2} = -E_j(\kappa \varepsilon_{i,i}^1 + \alpha_{ils} e_{ls,i}^1 + \beta E_i e_{ss,i}^1) - \beta E_n \varepsilon_{n,j}^1 - 2\varepsilon_{n,i}^1 \alpha_{nij}.$$

The approximation process could be continued in the same way further on.

The interpretation of equations (6) and (7) is very interesting. The right side of the first of them determines the electric charge created by the seismic focus; the second equation contains the additional dynamic force acting in the earthquake focus because of the electro-elastic coupling.

At present we shall find the exact solutions of equations (4) and (5). We will also confine ourselves, as it has been mentioned before, to the isotropic case only, thus we will neglect the part connected with the piezoelectricity leaving only the electrostrictive effects proportional to the square of the electric field.

Thus the initial equations have the following form:

$$\begin{aligned} \tau_{ij,i}^0 - E_j(-\kappa\varphi_{,ii} + \beta E_i e_{ss,i}) + \beta E_n \varphi_{,nj} + \rho f_j &= \rho \ddot{u}_j, \\ \varphi_{,ii} - 4\pi\beta E_i e_{ss,i} &= 0, \end{aligned} \tag{8}$$

where τ_{ij}^0 is the purely mechanical tensor. Introducing now the commonly used potentials

$$u_s = \xi_{,s} + \varepsilon_{slk}\eta_{k,l} \qquad \eta_{k,k} = 0,$$

$$\rho f_s = F_{,s} + \varepsilon_{slk}G_{k,l} \qquad G_{k,k} = 0,$$

where ε_{slk} is the skew-symmetric tensor, we obtain the following set of equations:

$$\lambda\xi_{,ssj} + 2\mu\xi_{,ssj} + \mu\varepsilon_{jlk}\eta_{k,ssl} + \kappa E_j\varphi_{,ll} - \beta E_j E_l \xi_{,ssl}$$

$$+ \beta E_s \varphi_{,sj} + F_{,j} + \varepsilon_{jlk}G_{k,l} = \rho\ddot{\xi}_{,j} + \varepsilon_{jlk}\ddot{\eta}_{k,l}, \tag{9}$$

$$\varphi_{,ii} - 4\pi\beta E_i \xi_{,ssi} = 0. \tag{10}$$

Equation (9) gives the following relations

$$(\lambda + 2\mu)\xi_{,ss} + (\kappa + \beta)E_s\varphi_{,s} - \beta E_s E_n \xi_{,sn} + F = \rho\ddot{\xi}, \tag{11}$$

$$\mu\eta_{l,ss} - \kappa E_j \varepsilon_{jLs}\varphi_{,s} + \beta E_j E_s \varepsilon_{jlk}\xi_{,sk} + G_l = \rho\ddot{\eta}_l. \tag{12}$$

In particular, equation (10) gives the relation:

$$\varphi - 4\pi\beta E_s \xi_{,s} = 0, \tag{13}$$

which allows us to obtain an equation for ξ from equation (11):

$$(\lambda + 2\mu)\xi_{,ss} + [4\pi\beta(\beta + \kappa) - \beta]E_s E_n \xi_{,sn} + F = \rho\ddot{\xi}.$$

To simplify the calculations let us suppose now, that the field E_i is directed along the z-axis: $(0, 0, E)$. In that case our last equation leads to the following relation:

$$(\lambda + 2\mu)\left(\Delta_{xy}\xi + \frac{\partial^2}{\partial z^2}\xi\right) + [4\pi\beta(\beta + \kappa) - \beta]E^2\frac{\partial^2}{\partial z^2} + F = \rho\ddot{\xi}$$

which, after a change of coordinates $xyz \to xyz'$, where

$$z' = \sqrt{\frac{\lambda + 2\mu}{\lambda + 2\mu + [4\pi\beta(\beta + \kappa) - \beta]E^2}}\, z \tag{14}$$

gives a usual equation for the elastic medium.

Taking into consideration the expression for φ, equation (12) gives:

$$\mu\eta_{l,ss} + \beta E_j E_n \varepsilon_{jlp}\xi_{,np} + G_l = \rho\ddot{\eta}_l.$$

If the field E_i is of the form $(0, 0, E)$, the above equations are now as follows

$$\mu\eta_{3,ss} + G_3 = \rho\ddot{\eta}_3,$$

$$\mu\eta_{1,ss} + \beta E^2\xi_{,23} + G_1 = \rho\ddot{\eta}_1,$$

$$\mu\eta_{2,ss} - \beta E^2\xi_{,13} + G_2 = \rho\ddot{\eta}_2.$$

These last equations are also typical for the elastic medium, with some additional forces $\beta E^2\xi_{,23}$ and $-\beta E^2\xi_{,13}$ distributed in space.

The interpretation of equation for the P-wave potential is much easier: the change of a linear scale takes place in the direction of the field E, as given by the equation (14). In accordance with that the stress field changes in the same way.

Thus the mechanical effect of a coupled electric field could be expressed as the change of a scale along that field. For example for an earthquake source described as the double dipole with moment we will obtain, on the plane diagram, the change of an angle between the nodal lines from a right angle to an acute one (Fig. 1): in real conditions this effect is probably very hard to observe. Much more important is the measurement of the electric field connected with an earthquake.

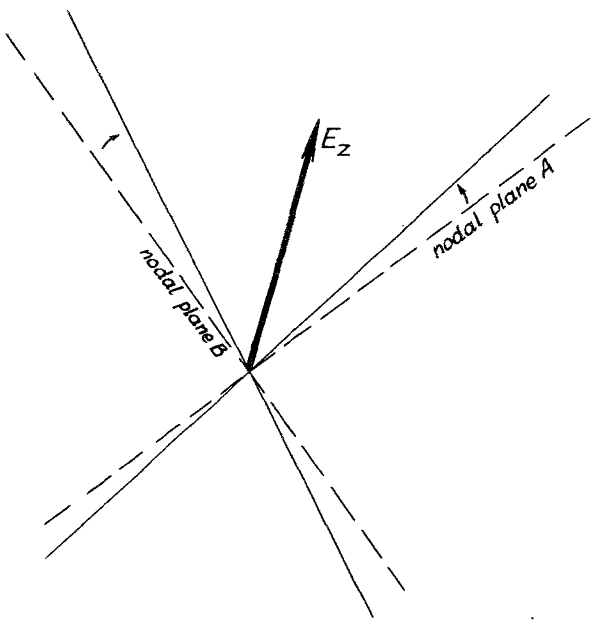

Figure 1
Change of the position of the nodal planes A and B.

The expression (13) gives us the potential of an electric field associated with deformations in an earthquake source, for the case of the electrostriction only. The full relation (6) including also the piezoelectric effects, allows us to compare the expressions describing the electric charge in dependence on the distance from an earthquake source both for the electrostriction and for the piezoelectric effects $(\alpha_{ils} e^1_{ls,i}, \beta E_i e^1_{ss,i})$. In that case the piezoelectric effect is averaged for an arbitrary crystal orientation.

The above presented theoretical model has been introduced with the general aim to take into account the piezoelectric and electrostrictive effects associated with earthquake source, so we will discuss now some constants and coefficients involved in this theory. Let us note first, that piezoelectric effects occurring in rocks have been studied mainly with the purpose of prospecting geophysics (IVANOV, 1940, 1949;

PARKHOMENKO, 1956, 1957, 1965; VOLAROVITCH and PARKHOMENKO, 1954; VOLAROVITCH *et al.*, 1959, 1962). It has been found that piezoelectric effect could be traced in rocks which include quartz and its magnitude is highest in granitic rocks (VOLAROVITCH and PARKHOMENKO, 1954), being of the order of few per cent of the same effect in pure quartz (PARKHOMENKO, 1957). It has also been noted, that piezoelectric effect in rocks could be the main reason of some electrical disturbances occurring during earthquakes, and that the thorough investigation of this effect could be useful during the determination of earthquake sources laying in granitic rock layers (PARKHOMENKO, 1957). As concerns the ratio of piezoelectric energy to mechanical energy, for most piezoelectric rocks this value is of the order of 10^{-3} and for granitic rocks it reaches even 10^{-2} (VOLAROVITCH *et al.*, 1962). This ratio could be used to estimate the value of electric charge created during the earthquake in piezoelectric rocks.

The electric potentials in granitic rocks could reach 10^{-2} V/m (SOBOLEV *et al.*, 1975). A more detailed discussion of piezoelectric data of rock materials, in connection with piezoelectric and electrostrictive phenomena associated with earthquakes, will be found in a survey paper by DMOWSKA (1977).

3. Electrokinetic effects connected with earthquakes

Before we construct the theoretical model of electrokinetic phenomena connected with earthquakes and caused by the diffusion of fluids into dilatant region, we would like to look critically at some hypotheses and assumptions connected with such processes. The first goal should be the dilatancy-diffusion hypothesis itself: though dilatancy is at present widely accepted as the mechanism responsible for certain effects observed before and during earthquakes (NUR, 1972; NUR and BOOKER, 1972; SCHOLZ *et al.*, 1973; ANDERSON and WHITCOMB, 1973; NUR, 1974, 1975), it is also argued, that the fluid-diffusion may be only one of several possible explanations of such effects (HANKS, 1974; BRADY, 1974, 1975; MYACHKIN *et al.*, 1975; STUART, 1974; STUART and DIETERICH, 1974; STUART, 1975; GARG, 1974; GARG *et al.* (in preparation), 1976). This limits possibility of occurrence of electrokinetic phenomena to some earthquakes only.

The second limitation is placed by the spatial extent of the dilatant region and its global permeability, caused by different types of pores, cracks and fractures. The present knowledge of these factors is rather speculative: the laboratory data show clearly, that the permeability of the laboratory samples is highly insufficient for the existence of the electrokinetic phenomena. However, it should be noted that the laboratory measurements concerning dilatancy are performed on the intact laboratory specimens and not always under proper conditions of stresses and temperature, therefore these results cannot be used directly to estimate the rock permeabilities under crustal conditions. On the other hand, it is common to assume, that rocks *in situ* are highly fractured and contain joints and cracks, and therefore their global permeability could be fairly large even when porosity itself is quite small. Existing field measurements (see, e.g., SNOW, 1968; BOARDMAN, 1970; DUNCAN *et al.*, 1972) seem to be in favor of that assumption and indicate that substantial permeability, sufficient for the existence of electrokinetic phenomena, may exist at depths of the order of a few kilometers or more. Therefore it is probably safe to conclude, that the electrokinetic phenomena could accompany some very shallow earthquakes, though the exact determination of the magnitude of this phenomenon, assuming any rock permeability, would be highly speculative.

Finally we would like to discuss briefly one more problem connected with electrokinetic phenomena induced by ground water flow associated with earthquakes. Accepting the physical description of the earthquake process as given by the dilatancy-diffusion theory with its consecutive stages (see, e.g., SCHOLZ *et al.*, 1973), one has to notice, that the magnitude of electrokinetic phenomena would probably depend on the liquid–vapor transition that takes place in the deeper parts of the crust. Thus it should be remembered, that the simple capillaric models with the full flow of liquid, used in general for purposes of physical chemistry in the case of earthquakes, could correspond to one stage only during the earthquake process. The more realistic description should be probably attained by the use of more complicated theoretical models of dilatancy-diffusion (as, e.g., proposed in GARG *et al.*, 1974; GARG *et al.*, 1975; GARG *et al.* (in preparation), 1976).

We will construct now a theoretical model of electrokinetic phenomena connected with an earthquake. We will consider a fluid-saturated porous rock, using the mechanical constitutive relations formulated by GARG *et al.* (1974).

The equations of motion for saturated elastic porous media consist of two separate equations of motion for solid and fluid phases (GARG *et al.*, 1974):

$$\rho'\ddot{u}_i' = \sigma_{ij,i}' + D(\dot{u}_i'' - \dot{u}_i'), \tag{1}$$

$$\rho''\ddot{u}_i'' = \sigma_{ij,i}'' - D(\dot{u}_i'' - \dot{u}_i'), \tag{2}$$

where superscripts (′) and (″) refer to the solid and liquid phases, respectively, and u_i is the displacement in the ith direction, σ_{ij} the stress tensor and ρ', ρ'' describe the partial densities, and σ_{ij}', σ_{ij}'' partial stresses:

$$\rho = \rho' + \rho'', \qquad \sigma_{ij} = \sigma_{ij}' + \sigma_{ij}'',$$

where ρ is the total mass per unit volume of composite, and

$$\rho' = (1 - \phi)\rho_r = n\rho_r, \tag{3}$$

$$\rho'' = \phi\rho_l = (1 - n)\rho_l, \tag{4}$$

$$D = (1 - n)^2 \frac{\rho v}{k}. \tag{5}$$

Here $\phi = 1 - n$ is the porosity, ρv is the dynamic fluid viscosity, k describes permeability and ρ_r, ρ_l are the rock and liquid densities, respectively.

Considering now the electrokinetic effect in the saturated elastic porous medium, equations of motion should be modified to the following form:

$$\left.\begin{aligned} \rho' \ddot{u}_i' &= \sigma'_{ij,i} + D(\dot{u}_i'' - \dot{u}_i') + \alpha E, \\ \rho'' \ddot{u}_i'' &= \sigma''_{ij,i} - D(\dot{u}_i'' - \dot{u}_i') - \alpha E, \end{aligned}\right\} \tag{6}$$

where αE describes the action of electrokinetic forces, E being the electric field.

Here

$$\alpha = (1 - n)\frac{\varepsilon\zeta}{\rho\nu} \tag{7}$$

where ζ is the electrokinetic potential and ε is the dielectric constant of the liquid.

The above equations should be completed by the general relations between the electric current i and fluid flow flux j as well as liquid pressure p and the electric field E,

$$i = \sigma E = \sigma^0 E^0 - \frac{\varepsilon\zeta}{\rho\nu}\operatorname{grad} p \tag{8}$$

$$j = \rho''(\dot{u}'' - \dot{u}') = -\alpha E - \frac{\rho''}{D}\operatorname{grad} p \tag{9}$$

where σ is a global conductivity.

Equations (8) and (9) are essentially the same as those given by MIZUTANI *et al.*, 1975, namely in the equation (8) (the Ohm's law) we assume that the electric field E is a global quantity consisting of the original electric field E^0 and the additional field E' caused by the electrokinetic effect. The second (equation 9) in the absence of electric field becomes simply the Darcy's law:

$$D(\dot{u}'' - \dot{u}') = -\operatorname{grad} p.$$

Further discussion is limited to the case when the new space created during dilatancy process is immediately filled up by liquid, thus the fluid flow is governed by the following condition ($\Delta V = 0$):

$$nu'_{i,i} + (1 - n)u''_{i,i} = 0. \tag{10}$$

Adding now the equations of motion for both phases (rock, and liquid), we obtain

$$\rho'\ddot{u}_i' + \rho''\ddot{u}_i'' = \sigma'_{ij,i} + \sigma''_{ij,i} = \sigma_{ij,i}. \tag{11}$$

The left-hand part defines here the average motion of composite, and we could write the general equation of motion in the presence of body forces F_i:

$$\rho\ddot{u}_i = \sigma_{ij,i} + \rho F_i. \tag{12}$$

Let us assume that the earthquake source acting in the fluid-saturated composite could be described by the point Love's force. We will denote the corresponding solution of the equation (12) by u^L.

Thus our basic solutions of equations (10) and (12) will be given by the following relations

$$\rho' u_i' + \rho'' u_i'' = \rho u_i^L, \tag{13}$$

$$n u_{i,i}' + (1 - n) u_{i,i}'' = 0. \tag{14}$$

Taking the divergence of the first equation we obtain $u_{i,i}''$ and $u_{i,i}'$:

$$u_{i,i}' = \frac{\rho u_{i,i}^L (1 - n)}{(1 - n)\rho' - n\rho''}, \tag{15}$$

$$u_{i,i}'' = \frac{\rho u_{i,i}^L n}{(1 - n)\rho' - n\rho''}. \tag{16}$$

The difference between relations (15) and (16) could be associated with Darcy's equation:

$$\dot{u}_{i,i}'' - \dot{u}_{i,i}' = \frac{\rho u_{i,i}^L}{n\rho'' - (1 - n)\rho'} = \frac{1}{D}\sigma_{ss,ii}'' - \frac{\alpha}{\rho''} E_{i,i}, \tag{17}$$

where

$$\operatorname{grad} p = -\sigma_{ss,i}''. \tag{18}$$

Relation (17) is the fundamental equation for the following discussion. Let us observe, that the left-hand side of that relation, connected with the Love's point-force solution, is the known value. The right-hand side of equation (17) consists of two parts: first, connected with a pore pressure gradient, and the second, describing the electrical effects connected with an earthquake in a porous medium. To estimate these effects from equation (17) we should find the expression $(1/D)\sigma_{ss,ii}''$.

One way of obtaining the pore pressure gradient is to find it from the field data. This way was followed by MIZUTANI *et al.* (1976); they estimated it as being between 1 and 10^2 bar/km in the dilatant focal region preceding or following the earthquake.

Another way of obtaining the expression $(1/D)\sigma_{ss,ii}''$ is to find it from the constitutive relations for fluid-saturated elastic porous media. Such relations have been developed by BIOT (1956), BIOT and WILLIS (1957); GARG and NUR (1973) and recently, by GARG *et al.* (in preparation, 1976). We will use here the relations presented by GARG and NUR (1973). The strain-displacement relations for the solid and liquid could be written as:

$$\left.\begin{aligned} e_{ij} &= \tfrac{1}{2}(u_{i,j} + u_{j,i}), \\ e &= e_{ii} = u_{i,i}, \qquad \hat{e}_{ij} = e_{ij} - \tfrac{1}{3} e \delta_{ij}, \end{aligned}\right\} \tag{19}$$

and the constitutive relations would be of the form:

$$\sigma_{ij}' = (\bar{a} e' + c e'')\delta_{ij} + 2\mu \hat{e}_{ij}', \tag{20}$$

$$\sigma_{ij}'' = (c e' + b e'')\delta_{ij} + 2\nu \dot{\hat{e}}_{ij}'', \tag{21}$$

where superscript (′) relates to the rock matrix and (″) to the liquid, and

$$\begin{aligned}\bar{a} &= nK_1(1 + b_1),\\ b &= (1 - n - nb_2)K_2,\\ c &= nK_1b_2 = -nK_2b_1,\\ b_1 &= a_1(1 - n)[1 - n - n(K_2/K_1)a_1]^{-1},\\ a_1 &= K/nK_1 - 1.\end{aligned}$$

Here K and μ denote the bulk and shear moduli of the porous solid, K_1 and K_2 are the intrinsic bulk moduli of the nonporous solid and the fluid one, respectively. Relations (20) and (21) are consistent with those used by GARG and NUR (1973) with one exception only, namely we have added the term $2\nu\dot{\hat{e}}''_{ij}$ in the equation (21), connected with the viscous resistance of the fluid, ν being the kinematic viscosity of the fluid.

From equation (21) we have

$$\sigma''_{ii} = 3(ce' + be'') = 3(cu'_{i,i} + bu''_{i,i}). \tag{22}$$

Inserting now relation (22) and (15–16) into equation (17), we obtain the formula describing the divergence of the global electric field connected with an earthquake:

$$E_{i,i} = -\frac{\rho}{\alpha}[n\rho'' - (1 - n)\rho^1]^{-1}\left\{\rho\dot{u}^L_{i,i} - \frac{3\rho u^L_{i,iss}}{D}[c(1 - n) - bn]\right\}. \tag{23}$$

The right-hand side describes here the effective electric charge density caused by electrokinetic processes connected with an earthquake. The electric field could be obtained from equation (23) by transforming it to the Poisson's equation for an electric potential.

Because of the very strong condition (10), the term $[(1 - n)\rho' - n\rho'']^{-1}$ could be theoretically infinite, but the existing estimations of porosity in dilatant regions and finite permeability show clearly that it will never occur. Therefore we can expect that a finite electric field exists associated with an earthquake.

One should also notice, that the solution u^L could be regarded as any possible solution of equation (12) and not only the Love's point solution. Thus equation (23) is the general relation describing the electrokinetic effect caused by deformation processes during an earthquake.

The existing estimations of magnitude of electrokinetic phenomena associated with earthquakes differ strongly.

The Japanese evaluations (MIZUTANI *et al.*, 1976) suggest, that the mechano-electric coupling is comparatively strong, i.e. that the electrokinetic phenomenon could constitute a relatively effective mechanism in creation of electric and magnetic anomalies accompanying earthquake processes. They adopt for E the value of 10^{-4} up to 10^{-1} V/m and grad $p = 10^{-3}$–10^{-1} kG/cm^2 m. On the other side, Russian estimations (SOBOLEV *et al.*, 1975) of the same parameters are much smaller, e.g. they estimate the electric field created by electrokinetic phenomena as low as 10^{-5} to 10^{-6} V/m. It is quite obvious that more field observations are required to evaluate better the role of these phenomena in earthquake process.

Acknowledgment

We would like to thank the anonymous reviewers for valuable comments.

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(Received 6th June 1976)